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HEAT AND MASS TRANSFER EFFECTS IN A REACTING NON-NEWTONIAN FLUID IN LAMINAR FLOW IN A VERTICAL TUBE

A THESIS

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by

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HEAT AND MASS TRANSFER EFFECTS IN A REACTING

NON-NEWTONIAN FLUID IN LAMINAR FLOW

IN A VERTICAL TUBE

Approved: ______________________

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Chairman

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NOMENCLATURE

This table contains the definitions of the symbols used throughout this work. It does not contain symbols defined and used locally within the body of this work. Dimensional variables are primed to distinguish them from dimensionless variables. Units are designated by Q for thermal units, M for mass units, L for length units, t for time and T for thermal degrees.

Symbol	Definition
C_i	expansion coefficient, defined by Equation IV-5.
$\hat{C}_p$	heat capacity, $\hat{C}'_p/\hat{C}'_{po}$ $\hat{C}'_p$ [=] Q/MT.
c_1, c_2	constants in density equation.
D	mean diffusivity, D'/D'_o D' [=] M/Lt.
E_a	energy of activation for reaction rate constant, $\frac{\Delta H'_a}{\mathcal{R}' T'_o}$.
E_v	energy of activation for flow consistency index, $\frac{\Delta H'_v}{\mathcal{R}'}\left(\frac{T'_w - T'_o}{T'_w T'_o}\right)$.
E_c	Eckert number, $\frac{U_o'^2}{C'_{po} T'_o}$.
F_c	free convection number, $\frac{\rho_{out}}{\rho_o}\left(\frac{\rho_{out}-\rho_o}{\rho_{out}+\rho_o}\right)\theta$.
Fr	Froude number, $\frac{U_o'^2}{g'_z\, 2R'}$.
g'_z	acceleration due to gravity in the z direction, g'_z [=] L/t^2.

Symbol	Definition
H_i	enthalpy of pure species i per mole, $H_i'/C_{po}'T_o'$ H' [=] Q/M.
$\bar{H}_i$	partial molal enthalpy of species i, $\bar{H}_i'/C_{po}'T_o'$ H' [=] Q/M.
$\Delta H_i'$	heat of reaction per mole of species i, $\Delta H_i'$ [=] Q/M.
$\Delta\hat{H}_i'$	heat of reaction per mass unit of species i, $\Delta\hat{H}_i'$ [=] Q/M.
$\Delta H_a'$	activation enthalpy for reaction rate constant, $\Delta H_a'$ [=] Q/M.
$\Delta H_v'$	activation enthalpy for flow consistency index, $\Delta H_v'$ [=] Q/M.
k	thermal conductivity, k'/k_o' k' [=] Q/LTt.
K_r	reaction rate constant, K_r'/K_{ro}' K_r' [=] t^{-1}.
K_v	flow consistency index, K_v'/K_{vo}' K_v' [=] M/Lt^{2-n}.
M_i	molecular weight of species i, M_i [=] M/Mole.
N_1, N_2	integers defined in Equation IV-7.
n	flow behavior index, dimensionless.
Nu_L	local Nusselt number defined by Equation A-31.
Nu_o	Nusselt number based on the entering temperature drop, defined by Equation A-39.
Nu_{AM}	arithmetic mean Nusselt number defined by Equation A-41.
Nu_{LN}	log mean Nusselt number defined by Equation A-43.
P	pressure, $P'/\rho_o'U_o'^2$ P' [=] $\frac{M}{Lt}$.
Pr	Prandtl number, $\frac{Cp_o'Kv_o'}{k_o'}\left(\frac{2R'}{U_o'}\right)^{1-n}$.

Symbol	Definition
R'	tube radius, R' [=] L.
r	tube radius, $r'/2R'$ r' [=] L.
R_i	function of r defined by Equation IV-5.
$\mathcal{R}'$	gas constant, $\mathcal{R}'$ [=] $\frac{Q}{MT}$.
Re	Reynolds number, $\frac{(2R')^n U_o'^{2-n} \rho_o}{K_{vo}'}$.
Sc	Schmidt number, $\frac{K_{vo}'}{D_o'} (\frac{2R}{U_o})^{1-n}$.
T	temperature, $\frac{T'}{T_o'}$ or $\frac{T'-T_w'}{T_o'-T_w'}$.
U'	average velocity, $\frac{U'}{U_o'}$ U' [=] L/t.
u	axial velocity, $\frac{u'}{U_o'}$ u' [=] L/t.
v	radial velocity, $\frac{v'Re}{U_o'}$ v' [=] L/t.
W_m	average mass fraction, dimensionless.
w	mass fraction, dimensionless.
x	radial distance defined by Equation IV-8.
z	axial distance, $z'/2R'Re$.
z^*	axial distance, $16(\frac{1+n}{1+3n}) \frac{\alpha}{Sc} z$.
Greek Symbols	
α	reaction rate parameter, $\frac{R'^2 K_{ro}' \rho_o'}{4D_o'}$.
β_i	coefficient in power series solution, defined by Equation IV-10.
ρ	density, ρ'/ρ_o' ρ' [=] $\frac{M}{L^3}$.
λ_i	eigenvalue.

Greek Symbols	Definition
θ	ratio of Reynolds number and Froude number, Re/Fr.
τ_{rz},	shear stress, τ_{rz}, [=] M/t^2L.
ξ	axial distance, $16(\frac{1+n}{1+3n})\frac{z}{Sc}$ or $16(\frac{1+n}{1+3n})\frac{z}{Pr}$.
η'	non-Newtonian viscosity, defined by Equation II-2, η' [=] M/Lt.

Subscripts

AM	arithmetic mean.
j	radial grid coordinate.
ℓ	axial grid coordinate.
L	local value.
LN	log mean value.
m	mean value.
o	entrance value.
w	wall value.

Superscript

^	denotes a quantity per mass unit as opposed to molal unit.

SUMMARY

Although non-Newtonian fluids occur often in the chemical industry, few studies have been reported in which the effects of the non-Newtonian characteristics of the fluid are considered in a realistic manner. In particular, the fields of heat and mass transfer accompanying a reacting non-Newtonian fluid have received little attention. The primary objective of this study was to consider the operation of a laminar flow, nonisothermal, tubular reactor. Secondary objectives were the study of heat transfer to a non-Newtonian fluid from a constant temperature wall and mass transfer to a non-Newtonian fluid from a constant composition wall.

The specific objective of this work was to solve the equations of continuity, motion, energy and diffusion for the situations described above. The physical properties were to be considered realistic functions of temperature and concentration, and the inertial terms in the equation of motion and the radial velocity terms in the equations of energy and diffusion were to be retained. The equations were simplified by a boundary-layer analysis to the Prandtl boundary-layer equations. The axial conduction of heat and diffusion of mass were neglected.

Before solutions to the complete equations were obtained, several simplified models were studied. For the tubular reactor problem, a "plug flow" model representing infinite radial diffusion and a "parabolic flow" model representing no radial diffusion were considered. These models showed that diffusion reduces the length of tube required

for a given conversion. These models were also extended to include varying density and heat effects. For a simple model for the heat-and mass-transfer problem, an extension of the Lévêque solution to non-Newtonian fluids is proposed.

Analytical solutions were obtained to the problems considered for the case of constant properties and fully developed flow. These solutions permit interpolation between the results of the limiting simplified models.

The solutions to the complete equations for variable properties and fully developed flow were determined numerically by a finite difference technique. The method consists of replacing the partial derivatives with finite difference approximations. This produces a system of simultaneous algebraic equations which must then be solved implicitly for the velocity, temperature and concentration profiles. Systems of equations similar to those used in this work have been shown to be stable and convergent by earlier workers. The accuracy of the scheme was checked by comparison of the results with the analytical results and the little experimental data available. Variable property heat transfer results agreed well with those of Wilkins (15). A step by step check of the energy equation showed the tubular reactor problem to be a much more stable problem numerically than the heat-transfer problem. Since the heat-transfer problem gave results in good agreement with those of Wilkins (15), the tubular reactor problem was felt to be stable and accurate.

The numerical results were correlated by the simple models discussed previously. Tables of results are presented which permit interpolation between the limiting simple models for constant properties and for varying density. The effect of a developing velocity profile was found to be important for

$$\frac{\alpha}{Sc} = \frac{R'^2 K'_r \rho'}{4\, K'_v} \left(\frac{U'_o}{2R'}\right)^{1-n} \leq n$$

The exothermic heat of reaction of reaction situation could be bounded by the isothermal "parabolic flow" model and the adiabatic "plug flow" model. The results were correlated by a heat of reaction,

$$\Delta\hat{H} = \frac{H_2}{M_2} - \frac{H_1}{M_1}$$

a parameter E_v representing the rate at which the reaction rate constant changes with temperature and defined by

$$K_r = \exp(E_v \frac{T - 1}{T})$$

a group controlling the heat-transfer rate

$$\frac{\alpha Pr}{Sc} = \frac{R'^2 K'_{ro} C'_{po} \rho'_o}{4\, k'_o}$$

and a length parameter

$$Z^* = 16\left(\frac{n+1}{3n+1}\right) \frac{\alpha}{Sc}\, z$$

$$= \left(\frac{n+1}{3n+1}\right) \frac{K'_r\, z'}{U'_o}$$

Concentration and temperature profiles and average concentration and temperature are presented for gases, Newtonian liquids and non-Newtonian fluids.

Heat-transfer results for non-Newtonian fluids are presented for constant properties and fully developed flow. The effect of varying rheological properties is also considered. The parameters here are E_v which is defined by

$$K_v = \exp(E_v\, T)$$

and

$$\xi = 16\left(\frac{n+1}{3n+1}\right) \frac{z}{Pe}$$

$$= 4\left(\frac{n+1}{3n+1}\right) \frac{k\ z}{R'^2 U'_o \rho'_o C'_{po}}$$

which is a length parameter.

CHAPTER I

INTRODUCTION

Many chemical engineering operations require the solution to problems associated with the individual or combined effects of fluid flow, heat transfer and mass transfer. One approach to such problems consists of the mathematical solutions to the general equations of continuity, motion, energy and diffusion. However, for complicated problems involving variable properties and developing flows, analytical solutions are practically impossible. Although numerical methods implemented on electronic computers are less rigorous, many useful results can be obtained by these techniques. This study was a numerical investigation of three problems involving a non-Newtonian fluid flowing in laminar flow in a vertical tube. These problems are:

(1) A tubular reactor.

(2) Heat transfer from a constant temperature wall.

(3) Mass transfer from a constant composition wall.

Homogeneous Tubular Reactor

The primary problem was that of a fluid reacting with a homogeneous first order reaction. Fuller (1) has studied this problem for turbulent flow of Newtonian fluids and discussed most of the previous work. However, no studies have been reported on this topic for laminar flow of non-Newtonian fluids and only a few have been reported for Newtonian fluids. These are summarized below.

Bosworth (2) was the first to consider the effects of diffusion on conversion by a complicated but intuitive argument. Although the results are of great value, they do not represent the solution to the diffusion equation.

Lauwerier (3) considered the diffusion equation for the case of constant physical properties and fully developed flow, neglecting axial diffusion and radial convection, and determined the form of the analytical solution. Wissler and Schechter (4) completed this problem with the determination of eigenvalues, expansion coefficients and norms.

Cleland and Wilhelm (5), using an electronic computer, solved the diffusion equation numerically and compared the results with experimental data. This work showed that free-convection effects based on concentration and temperature differences could be significant and indicated a need for variable property solutions. Vignes and Trambouze (6), studying a second order reaction, attacked the problem in almost the same manner as Cleland and Wilhelm (5) and obtained similar results.

Ulrichson and Schmitz (7) studied the effect of developing flow on homogeneous reaction by assuming the approximate velocity profiles of Langhaar (8). This technique, although quite valuable, cannot be extended to include variable properties or non-Newtonian fluids.

The effect of axial diffusion on conversion was studied experimentally by Dickens et al. (9) and analytically by Walker (10). The results indicated that this effect is unimportant except for very slow flows.

The only reported works which consider energy effects are those of Chambré (11, 12). His earlier paper discusses the "plug flow" model which is discussed in the third chapter of this work. The later paper considers the more involved problem but the assumptions used severely limit the usefulness of the equations. Actually the problem is merely reduced to a Sturm-Liouville problem which must be solved for each case.

The objectives of this work were to obtain the analytical solution for the general power-law fluid and to develop numerical solutions to include the variable property, developing flow cases.

Heat Transfer from a Constant Temperature Wall

The second problem studied was that of heat transfer from a wall. The analytical solution, assuming constant properties and fully developed flow, was first reported by Graetz (13) for Newtonian fluids. For many practical situations these assumptions are not valid and attempts have been made to remove these restrictions. Lee (14) has summarized most of these attempts and has presented a numerical solution for variable physical properties. Wilkins (15) has extended this to include the inertial terms in the equation of motion and the radial velocity term in the energy equation. This work is valid in the hydrodynamic entrance region of the tube as well as in the thermal entrance region.

Non-Newtonian fluids have only recently received much attention. Lyche and Bird (16) have extended the Graetz solution to the special case of power-law fluids of n = 0.5 and n = 0.2. Pigford (17) has extended the Lévêque solution to non-Newtonian fluids.

Metzner, Vaughan and Houghton (18) have presented experimental data and correlated it to within 13.5 per cent. The physical properties for aqueous solutions of Carbopol and sodium carboxymethylcellulose

(DM') are given as functions of temperature.

Craig (19) has also obtained a large amount of experimental data. He compared this to a numerical solution of the equations of motion and energy in which only the rheological properties were allowed to vary and fully developed flow was assumed. The mean deviation between calculated and experimental results was 7 per cent.

Lemmon (20) has numerically solved the equations of motion and energy for the developing flow case considering the rheological properties to be functions of temperature. However, he did not cover a wide range of conditions.

The objectives of this work were to obtain an analytical solution for the case of a general power-law fluid and to develop numerical solutions without the restriction of fully developed flow or constant properties.

Mass Transfer from a Soluble Wall

The third problem considered was that of mass transfer from a wall. This study was restricted to isothermal conditions and low mass transfer rates so that the radial velocity was assumed to be negligible at the wall. This problem is very similar mathematically to the heat-transfer problem, and indeed if constant properties and fully developed flow are assumed, the heat-transfer solution can be used for the mass-transfer problem merely by substituting the Schmidt number for the Prandtl number and concentration for temperature.

Linton and Sherwood (21) studied the diffusion of acids from soluble tube walls into a stream of water. This work indicated good agreement with the Graetz solution. Saunders (22) studied the diffusion of acids from soluble tubes into non-Newtonian fluids. The results were correlated by the Lévêque equation corrected with a term involving the viscosity of the non-Newtonian fluid divided by the viscosity of the solvent.

Since non-Newtonian fluids involve very low concentrations, variable property solutions are probably not highly significant. The purpose of this work was to obtain solutions to the diffusion equation which are valid for low concentrations.

Physical Properties

In a study of this type it is important to keep in mind the range of interest of the physical properties and the various parameters such as the Reynolds number, the Prandtl number and the Schmidt number. Accordingly, a literature search was made to obtain this information with special emphasis being placed on how these properties vary with temperature and concentration.

Although non-Newtonian fluids are of many diverse types such as polymer melts and solutions and aqueous solutions, suspensions, and slurries, primary interest here was placed on polymer melts and aqueous solutions of cellulosic polymers.

Non-Newtonian fluids are characterized by their rheological behavior and, although this study assumed a power-law relationship between shear stress and shear rate, it must be remembered that real fluids follow this relationship only approximately. Many non-Newtonian fluids have elastic properties and/or time-dependent properties which are not adequately described by this simple model. However, many engineering problems can be studied successfully using this assumption and many rheological studies are available, both for polymer melts (23, 24) and aqueous solutions of cellulosic polymers (18, 19), from which the necessary properties can be characterized as functions of temperature. Studies of concentration effects are limited (25).

Density measurements (26) and heat capacity measurements (26, 27) have been reported for polymer melts as functions of temperature. Similar measurements on aqueous solutions (18, 19) have indicated these properties to be very close to those of water.

Thermal conductivity measurements on polymer melts (25, 28) indicate little change with temperature but do indicate a low value of thermal conductivity approaching that of an insulating material. Measurements on aqueous solutions (28) have indicated the thermal conductivity to be equal to that of water, while others (18) have determined this value to be as much as 25 per cent below that of water.

Heats of reaction can be calculated by standard methods. Typical values for heats of polymerization (25) have been reported.

Values of diffusivity are the most difficult to obtain. Experimental values are tabulated as functions of temperature for gases (29), and as functions of temperature and concentration for certain organic liquids (30). However, only a few values are available (31, 22) for non-Newtonian fluids which are of primary interest.

The range of interest for the various parameters can be determined by combining values of the physical properties. Ranges of parameters used in this study are:

Parameter	Range	
Re	100-2200	
Pr	0.7-1.0	Gases
	5-100	Liquids
	10-1000	Non-Newtonian Fluids
Sc	0.7-2.5	Gases
	800-1200	Liquids
	10,000	Non-Newtonian Fluids

CHAPTER II

DEVELOPMENT OF EQUATIONS

In this chapter the physical situation is described in detail. The general equations of continuity, motion, energy, and diffusion are simplified and the assumptions are discussed.

Physical Description

A non-Newtonian fluid is flowing in laminar flow in a vertical circular tube and reacting in a homogeneous reaction. It can produce or absorb heat and exchange energy with its surroundings. At the tube inlet the velocity profile is either flat or parabolic and the fluid temperature is some constant value T_o. The tube wall temperature is maintained at a constant value T_w which can be equal to or different from T_o.

Mathematical Description

Tubular Reactor

The flow is considered to be steady and axially symmetric. In addition the boundary-layer assumptions are made. Applicability of the boundary-layer assumptions to non-Newtonian fluids has been discussed by Acrivos (32), Schowalter (33), and Collins (34). With these assumptions the z-component of the equation of motion becomes

$$\rho'\left(v'\frac{\partial u'}{\partial r'} + u'\frac{\partial u'}{\partial z'}\right) = -\frac{dP'}{dz'} - \frac{1}{r'}\frac{\partial}{\partial r'}(r'\ \tau_{rz'}) + \rho' g' \qquad \text{II-1}$$

For isotropic, non-Newtonian fluids the shear stress is given by

$$\tau_{rz'} = - \eta' \frac{\partial u'}{\partial r'} \qquad \text{II-2}$$

where η' is a scalar function of temperature, composition and the velocity field. The equation of motion then becomes

$$\rho'(v'\frac{\partial u'}{\partial r'} + u'\frac{\partial u'}{\partial z'}) = - \frac{dP'}{dz'} + \frac{1}{r'} \frac{\partial}{\partial r'}(r' \eta' \frac{\partial u'}{\partial r'}) + \rho' g_z' \qquad \text{II-3}$$

The empirical functional relationship used in this work between η' and the velocity field is the power-law model. Thus

$$\eta' = - K_v' \left|\frac{\partial u'}{\partial r'}\right|^{n-1} \qquad \text{II-4}$$

where K_v' and n are functions of temperature and composition alone. For n=1 the fluid is Newtonian and K_v' reduces to the Newtonian viscosity.

Casting the equation in non-dimensional form yields

$$\rho(v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z}) = - \frac{dP}{dz} + \eta \frac{\partial^2 u}{\partial r^2} + (\frac{\eta}{r} + \frac{\partial \eta}{\partial r}) \frac{\partial u}{\partial r} + \theta\rho \qquad \text{II-5}$$

Considering the enthalpy to be a function of temperature and composition, the energy equation (Equation E, Table 18.3-1, Bird (35)) becomes

$$\rho' C_p'(v' \frac{\partial T'}{\partial r'} + u' \frac{\partial T'}{\partial z'}) = \frac{1}{r'} \frac{\partial}{\partial r'}(r' k' \frac{\partial T'}{\partial r'}) + \sum_{i=1}^{2} \frac{1}{r'} \frac{\partial}{\partial r'}(r' \frac{\bar{H}_i'}{M_i'} D_i' \frac{\partial W_i'}{\partial r'})$$

$$- \rho' \sum_{i=1}^{2} \frac{\bar{H}_i'}{M_i'} (v' \frac{\partial W_i'}{\partial r'} + u' \frac{\partial W_i'}{\partial z'})$$

$$+ K_v' \left[(\frac{\partial u'}{\partial r'})^2\right]^{\frac{n+1}{2}} \qquad \text{II-6}$$

where the axial conduction of heat and the axial diffusion of mass have been neglected and the reaction mixture is limited to two components. Heat effects associated with pressure forces in the term

$$u' \frac{\partial P'}{\partial z'}$$

are also neglected. The last term in Equation II-6 represents viscous dissipation. If the equation is cast into non-dimensional form, the summation terms expanded and the relation

$$w_1 + w_2 = 1 \qquad \text{II-7}$$

used, the result is

$$\rho\hat{C}_p\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = \frac{1}{Pr}\left(k\frac{\partial^2 T}{\partial r^2} + \left(\frac{k}{r} + \frac{\partial k}{\partial r}\right)\frac{\partial T}{\partial r}\right)$$
$$- \frac{1}{Sc}\left(\Delta\hat{\bar{H}}\left(\frac{\partial^2 w}{\partial r^2} + \left(\frac{D}{r} + \frac{\partial D}{\partial r}\right)\frac{\partial w}{\partial r}\right) + D\frac{\partial w}{\partial r}\frac{\partial \Delta\hat{\bar{H}}}{\partial r}\right)$$
$$+ \Delta\hat{\bar{H}}\,\rho\left(v\frac{\partial w}{\partial r} + u\frac{\partial w}{\partial z}\right) + Ec\,K_v\left[\left(\frac{\partial u}{\partial r}\right)^2\right]^{\frac{n+1}{2}} \qquad \text{II-8}$$

where

$$\Delta\hat{\bar{H}} = \left(\frac{\bar{H}_2}{M_2} - \frac{\bar{H}_1}{M_1}\right) w_o \qquad \text{II-9}$$

Since data on the partial molal enthalpies is limited, the enthalpies of the pure components are usually used. For this situation $\Delta\hat{\bar{H}}$ becomes $\Delta\hat{H}$ which is the heat of reaction per pound of component A reacted. Although this assumption is not really justified, the use of a mean value of $\Delta\hat{H}$ would probably represent the data adequately. Numerical solutions allowing $\Delta\hat{H}$ to vary with composition and temperature would present no problems if sufficient data were available to characterize

$\Delta\hat{H}$ as functions of temperature and composition.

The diffusion equation, assuming a first order homogeneous reaction and no axial diffusion, becomes

$$\rho'\left(v' \frac{\partial w'}{\partial r'} + u' \frac{\partial w'}{\partial z'}\right) = D' \frac{\partial^2 w'}{\partial r'^2} + \left(\frac{D'}{r'} + \frac{\partial D'}{\partial r'}\right) \frac{\partial w'}{\partial r'} - K'_r \rho' w' \qquad \text{II-10}$$

In non-dimensional form this equation becomes

$$\rho\left(v \frac{\partial w}{\partial r} + u \frac{\partial w}{\partial z}\right) = \frac{1}{Sc}\left(\frac{\partial^2 w}{\partial r^2} + \left(\frac{D}{r} + \frac{\partial D}{\partial r}\right) \frac{\partial w}{\partial r} - 16\alpha\rho K_r w\right) \qquad \text{II-11}$$

The three equations -- the Equation of motion II-5, the Equation of energy II-8, and the Equation of diffusion II-11 -- are to be solved in conjunction with the equation of continuity

$$\frac{1}{r} \frac{\partial \rho r v}{\partial r} + \frac{\partial \rho u}{\partial z} = 0 \qquad \text{II-12}$$

and the overall mass balance

$$\int_0^{\frac{1}{2}} \frac{\partial \rho r u}{\partial z}\, dr = \text{constant} \qquad \text{II-13}$$

to determine the values of u, v, w, T and P. The boundary conditions for this system of equations are

I $\quad z = 0 \qquad K_v = \rho = C_p = k = D = K_r = 1 \; (0 \le r \le \frac{1}{2}) \; P = v = 0,$

$T = 1$

Either $\quad$ a. $u = \left(\frac{3n+1}{n+1}\right)\left(1 - (2r)^{\frac{n+1}{n}}\right)$

or $\quad$ b. $u = 1$

II $\quad r = 0 \qquad \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = v = 0 \quad \frac{\partial w}{\partial r} = 0$

III $\quad r = \frac{1}{2} \qquad v = u = 0,\; T = \frac{T_w}{T_o} \quad \frac{\partial w}{\partial r} = 0$

Some of the dimensionless variables used in these equations are listed below. A complete listing may be found in the nomenclature section.

$$r = \frac{r'}{2R'} \qquad z = \frac{z'}{2R'Re} \qquad P = \frac{P'}{\rho' U_o'^2}$$

$$u = \frac{u'}{U_o'} \qquad v = \frac{v'Re}{U_o'} \qquad T = \frac{T'}{T_o'}$$

$$Re = \frac{(2R')^n U_o'^{2-n} \rho_o'}{K_{vo}'} \qquad Fr = \frac{U_o'^2}{2R' g_z'}$$

$$Sc = \frac{K_{vo}'}{D_o'} \left(\frac{2R'}{U_o'}\right)^{1-n} \qquad Pr = \frac{\hat{C}_{po}' K_{vo}'}{k_o'} \left(\frac{2R'}{U_o'}\right)^{1-n}$$

$$\alpha = \frac{K_{ro}' R' \rho_o'}{4D_o'} \qquad Ec = \frac{U_o'^2}{C_{po}' T_o'}$$

<u>Heat Transfer from a Constant Temperature Wall</u>

For n = 1 (Newtonian fluid) this is the well-known Graetz problem. The fluid is not reacting and the diffusion equation is not required. For this situation the equation of motion and energy for non-Newtonian fluids becomes

$$\rho\left(v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z}\right) = - \frac{dP}{dz} + \eta \frac{\partial^2 T}{\partial r^2} + \left(\frac{\eta}{r} + \frac{\partial \eta}{\partial r}\right) \frac{\partial u}{\partial r} + \theta \rho \qquad \text{II-14}$$

and

$$\rho \hat{C}_p \left(v \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial z}\right) = \frac{1}{Pr} \left(k \frac{\partial^2 T}{\partial r^2} + \left(\frac{k}{r} + \frac{\partial k}{\partial r}\right) \frac{\partial T}{\partial r}\right) \qquad \text{II-15}$$

where

$$T = \frac{T' - T_o'}{T_w' - T_o'}$$

and viscous dissipation has been neglected.

The boundary conditions are

I $\quad z = 0 \qquad K_v = \rho = 1 \qquad (0 \le r \le \frac{1}{2})$

$$v = T = 0$$

Either a. $u = (\frac{3n+1}{n+1})(1 - (2R)^{\frac{n+1}{n}})$

or b. $u = 1$

II $\quad r = 0 \qquad v = \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0$

III $\quad r = \frac{1}{2} \qquad u = v = 0, \ T = 1$

Mass Transfer from a Wall

This problem is similar to the heat-transfer problem except that the wall is maintained at constant composition. This study was limited to isothermal conditions. Therefore, the equations of motion and diffusion are to be solved with special emphasis on the entrance region. The equation of motion retains the same form as Equation II-5 and the diffusion equation becomes

$$(v \frac{\partial w}{\partial r} + u \frac{\partial w}{\partial z}) = \frac{1}{Sc} (D \frac{\partial^2 w}{\partial r^2} + (\frac{D}{r} + \frac{\partial D}{\partial r}) \frac{\partial w}{\partial r}) \qquad \text{II-16}$$

The boundary conditions are

I $\quad z = 0 \quad D = Kv = \rho = 1 \qquad (0 \le r \le \frac{1}{2})$

$$v = w = 0$$

Either a. $u = (\frac{3n + 1}{n + 1})(1 - (2r)^{\frac{n+1}{n}})$

or b. $u = 1$

II $\quad r = 0 \qquad v = \frac{\partial u}{\partial r} = \frac{\partial w}{\partial r} = 0$

III $\quad r = \frac{1}{2} \qquad v = u = 0, \ w = 1$

CHAPTER III

SIMPLIFIED MODELS

Before proceeding to the solution of the general equations, it is worthwhile to consider two simple models of the tubular reactor. These models usually establish upper and lower bounds for the solutions to the complete problem. The first model, termed the "plug flow" model, assumes that diffusion is rapid compared to the reaction and that the concentration is uniform across the tube. The second model, termed the "parabolic flow" model, assumes that diffusion is negligible and that the concentration profile is established by the velocity profile which exists in the tube. This model predicts zero concentration at the wall and a maximum concentration at the tube axis.

The equations for these models may be obtained by deleting the less important terms from the general equations. Heat exchange with the environment is accounted for by a heat transfer coefficient. The equations become

$$\rho u \frac{\partial w}{\partial z} = - \frac{16}{Sc} \alpha K_r \rho w \qquad \text{III-1}$$

$$\rho u \frac{\partial T}{\partial z} = - \Delta\hat{H} \rho u \frac{\partial w}{\partial z} - \frac{8Nu}{Pr} (T - 1) \qquad \text{III-2}$$

where the last term in the energy equation represents the heat exchange with the environment. Defining

$$z^* = 16\left(\frac{n + 1}{3n + 1}\right) \frac{\alpha}{Sc} z \qquad \text{III-3}$$

gives

$$\rho u \frac{\partial w}{\partial z^*} = - \left(\frac{3n + 1}{n + 1}\right) K_r \rho w \qquad \text{III-4}$$

and

$$\rho u \frac{\partial T}{\partial z^*} = - \Delta\hat{H} \rho u \frac{\partial w}{\partial z^*} - 2\left(\frac{n + 1}{3n + 1}\right) \frac{Nu\ Sc}{\alpha\ Pr} (T - 1) \qquad \text{III-5}$$

The "plug flow" model assumes

$$u = 1 \qquad \text{III-6}$$

while the "parabolic flow" model assumes

$$u = \left(\frac{3n + 1}{n + 1}\right)\left(1 - (2r)^{\frac{n+1}{n}}\right) \qquad \text{III-7}$$

Constant Properties

For constant properties only the diffusion equation is of interest. The "plug flow" model reduces to

$$\frac{\partial w}{\partial z^*} = - \left(\frac{3n + 1}{n + 1}\right) w \qquad \text{III-8}$$

and the solution is

$$w = \exp\left(- \left(\frac{3n + 1}{n + 1}\right) z^*\right) \qquad \text{III-9}$$

The "parabolic flow" model becomes

$$\left(1 - (2r)^{\frac{n+1}{n}}\right) \frac{\partial w}{\partial z^*} = - w \qquad \text{III-10}$$

and the solution is

$$w = \exp\left(\frac{-z^*}{\left(1 - (2r)^{\frac{n+1}{n}}\right)}\right) \qquad \text{III-11}$$

The average concentration is found by an integration across the tube

$$W = 8\left(\frac{3n + 1}{n + 1}\right) \int_0^{\frac{1}{2}} r\left(1 - (2r)^{\frac{n+1}{n}}\right) w \, dr \qquad \text{III-12}$$

For Newtonian fluids Cleland and Wilhelm (5) have shown that this integral can be expressed in terms of the exponential integral, a tabulated function. For other values of n the integration can be performed numerically. A comparison of the two models for several values of n is presented in Table 4 of Appendix D.

Varying Density

If a linear variation of density with concentration is assumed

$$\rho = c_1 + c_2 w \qquad \text{III-13}$$

where

$$c_1 + c_2 = 1 \qquad \text{III-14}$$

then the diffusion equation for the "plug flow" model becomes

$$\frac{\partial w}{\partial z^*} = -\left(\frac{3n + 1}{n + 1}\right)(c_1 + c_2 w) w \qquad \text{III-15}$$

The solution to this equation is found to be

$$z^* = \frac{(1 + n)}{c_1(1 + 3n)} \ln \frac{c_1 + c_2 w}{w} \qquad \text{III-16}$$

$$w = \frac{c_1}{\exp\left(\left(\frac{3n + 1}{n + 1}\right) c_1 z^*\right) - c_2} \qquad \text{III-17}$$

The "parabolic flow" model yields a solution as follows:

$$z^* = \frac{(1 - (2r)^{\frac{n+1}{n}})}{c_1} \ln \frac{c_1 + c_2 w}{w} \qquad \text{III-18}$$

$$w = \frac{c_1}{\exp\left(\dfrac{c_1 z^*}{(1 - (2r)^{\frac{n+1}{n}})}\right) - c_2} \qquad \text{III-19}$$

The average concentration is found by a straightforward numerical integration across the tube. A comparison of the two models is given in Tables 5 and 6 of Appendix D.

Adiabatic Flow

For this case the energy equation can be integrated to give

$$T = 1 - \Delta\hat{H}\,(1 - w) \qquad \text{III-20}$$

Assuming that the variation of K_r can be expressed by

$$K_r = \exp\left(\frac{E_a(T - 1)}{T}\right) \qquad \text{III-21}$$

The solutions become

$$z^* = \left(\frac{n + 1}{3n + 1}\right) \int_1^w \frac{dw}{w \exp\left(\dfrac{\Delta\hat{H}E_a(1 - w)}{1 - \Delta\hat{H}(1 - w)}\right)} \qquad \text{III-22}$$

for the "plug flow" model and

$$z^* = (1 - (2r)^{\frac{n+1}{n}}) \int_1^w \frac{dw}{w \exp\left(\dfrac{\Delta\hat{H}E_a(1 - w)}{1 - \Delta\hat{H}(1 - w)}\right)} \qquad \text{III-23}$$

for the "parabolic flow" model. The average concentration is found by a trial and error integration of Equation III-19. A comparison of the two models can be made from the results presented in Tables 7 and 8 of Appendix D.

General Flow

Solutions to Equations III-4 and III-5 have been presented for limiting values of $\frac{Nu\ Sc}{\alpha\ Pr}$ approaching infinity (isothermal flow) and approaching zero (adiabatic flow). The solution of these equations for intermediate values of $\frac{Nu\ Sc}{\alpha\ Pr}$ for the "parabolic flow" model would present a number of problems. However, solution to the "plug flow" case is simple and several solutions are given in Tables 9, 10 and 11 of Appendix D.

Heat and Mass Transfer

A simplified model for the heat- and mass-transfer problems is the well-known Lévêque solution. This solution has been extended to include non-Newtonian fluids by Pigford (17). The results can be summarized by

$$T_m = 1.615 \left(\frac{1 + 3n}{4n}\right)^{1/3} \left(\frac{1 + 3n}{2 + 2n}\right)^{2/3} \xi^{2/3} \qquad \text{III-24}$$

$$Nu_{AM} = 3.23 \left(\frac{1 + n}{2n}\right)^{1/3} \xi^{-1/3} \qquad \text{III-25}$$

CHAPTER IV

ANALYTICAL SOLUTIONS

In this chapter analytical solutions are developed and the results compared with numerical solutions. Analytical solutions for the three problems considered in this study can be obtained for the case of a general power-law fluid flowing in fully developed flow and having constant properties. For these assumptions the equation of motion can be uncoupled from the equations of energy and diffusion and the velocity profile determined to be

$$u = \left(\frac{3n + 1}{n + 1}\right)\left(1 - (2r)^{\frac{n+1}{n}}\right) \qquad \text{IV-1}$$

Homogeneous Tubular Reactor

The diffusion equation under these assumptions becomes

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - Sc\, u \frac{\partial w}{\partial z} - 16\alpha w = 0 \qquad \text{IV-2}$$

with the boundary conditions

I	$z = 0$	$w = 1$
II	$r = 0$	$\frac{\partial w}{\partial r} = 0$
III	$r = \frac{1}{2}$	$\frac{\partial w}{\partial r} = 0$

Substituting Equation IV-1 into Equation IV-2 and defining

$$\xi = \frac{16z}{Sc\left(\frac{3n + 1}{n + 1}\right)} \qquad \text{IV-3}$$

gives

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - 16(1 - (2r)^{\frac{n+1}{n}})\frac{\partial w}{\partial \xi} - 16\alpha w = 0 \qquad \text{IV-4}$$

Proposing a solution of the form

$$w = \sum_{I=1}^{\infty} C_i \exp(-\lambda_i \xi) R_i(r) \qquad \text{IV-5}$$

gives

$$R_i'' + \frac{1}{r} R_i' + 16(\lambda_i(1 - (2r)^{\frac{n+1}{n}}) - \alpha) R_i = 0 \qquad \text{IV-6}$$

where the primes indicate differentiation with respect to r. In order to solve this equation it will be assumed that $\frac{n+1}{n}$ is a rational number, i.e., it can be expressed as a ratio of two integers.

$$\frac{n + 1}{n} = \frac{N_1}{N_2} \qquad \text{IV-7}$$

Making the substitution

$$x = (2r)^{\frac{1}{N_2}} \qquad \text{IV-8}$$

Equation IV-6 becomes

$$R_{ix}'' + \frac{1}{x} R_{ix}' + 4N_2^2 x^{2(N_2-1)}(\lambda_i(1 - x^{N_1}) - \alpha) R_{ix} = 0 \qquad \text{IV-9}$$

where the sub x denotes differentiation with respect to x. Assuming a power series solution to Equation IV-9 of the form

$$R = \sum_{k=0}^{\infty} \beta_{k+1} x^k \qquad \text{IV-10}$$

gives the recurrence relation

$$\beta_k = \frac{4N_2^2((\alpha - \lambda_i)\beta_{k-2N_2} + \lambda_i \beta_{k-2N_2 - N_1})}{(k-1)^2} \qquad k \geq 3 \qquad \text{IV-11}$$

$$\beta_k = 0 \qquad k \leq 0 \qquad \text{IV-12}$$

where β_1 is arbitrarily set equal to unity and β_2 is equal to zero by boundary condition at $r = 0$.

To determine the infinite set of positive eigenvalues, the boundary condition at $r = \frac{1}{2}$ is used.

$$\sum_{k=1}^{\infty} k \ \beta_{k+1} = 0 \qquad \text{IV-13}$$

In order to determine the expansion coefficients, C_i, the following orthogonality relation is used

$$\int_0^{\frac{1}{2}} r(1 - (2r)^{\frac{n+1}{n}}) \ R_m R_n \ dr = 0 \qquad m \neq n \qquad \text{IV-14}$$

Using Equation IV-14 and the boundary condition at $z = 0$, the expansion coefficients are found to be

$$C_i = \frac{4 \int_0^{\frac{1}{2}} r(1 - (2r)^{\frac{n+1}{n}}) \ R_i \ dr}{\overline{N}_i} \qquad \text{IV-15}$$

where the norm, $\overline{N}_i$, is defined by

$$\overline{N}_i = 4 \int_0^{\frac{1}{2}} r(1 - (2r)^{\frac{n+1}{n}}) \ R_i^2 \ dr \qquad \text{IV-16}$$

The average concentration is then found to be given by

$$W_M(\xi) = 2\left(\frac{3n + 1}{n + 1}\right) \sum_{i=1}^{\infty} C_i^2 \ \overline{N}_i \ \exp(-\lambda_i \xi) \qquad \text{IV-17}$$

This solution has been reported for Newtonian fluids by Lauwerier (3) and Wissler (4).

Heat Transfer from a Constant Temperature Wall

The energy equation under the assumptions of constant properties and fully developed flow becomes

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \Pr\, u\,\frac{\partial T}{\partial z} = 0 \qquad \text{IV-18}$$

with the boundary conditions

I	$z = 0$	$T = 1$
II	$r = 0$	$\frac{\partial T}{\partial r} = 0$
III	$r = \frac{1}{2}$	$T = 0$

Defining

$$\xi = \frac{16z}{\Pr\,\left(\frac{3n + 1}{n + 1}\right)} \qquad \text{IV-19}$$

converts Equation IV-18 into the same form as Equation IV-4 with $\alpha = 0$. The solution to this equation and boundary conditions proceeds exactly as the solution to the previous equation with $\alpha = 0$ and the equation for determination of eigenvalues changed to

$$\sum_{k=0}^{\infty} \beta_{k+1} = 0 \qquad \text{IV-20}$$

This solution has been reported by Lyche (16) for the special cases $n = 1$, $n = 0.5$ and $n = 0.2$.

Mass Transfer from a Soluble Wall

The solution to this problem is the same as the solution to the heat-transfer problem if Sc is substituted for Pr and concentration for temperature.

Results

Tubular Reactor

Eigenvalues, expansion coefficients and norms were calculated for n = 0.2, 0.5, 1, and 1.5 for values of α from 0.25 to 25. The results are presented in Table 12 of Appendix E. These results may be used with Equation IV-17 to calculate the average concentration as a function of distance.

Eigenvalues for n=1 have been reported by Wissler and Schechter (4) and are in good agreement with those presented here. Comparison of the radial concentration profiles with those of Cleland and Wilhelm (5) and the numerical results presented in this work is given in Figure 1. The average axial concentration profiles are compared with numerical solutions in Figure 2. Comparison of calculated results and the experimental results of Cleland (5) is found in Figure 3.

The analytical solutions presented here are valid for α less than about 25 and for Z^* greater than about 0.2. Attempts to extend these solutions to higher values of α and lower values of Z^* by the determination of more eigenvalues was unsuccessful due to round-off errors. However, in Figure 4 it is apparent that the solution for α = 25 differs from the simple model, "parabolic flow" model, by only a few per cent and that larger values of α are less important for the constant property solutions.

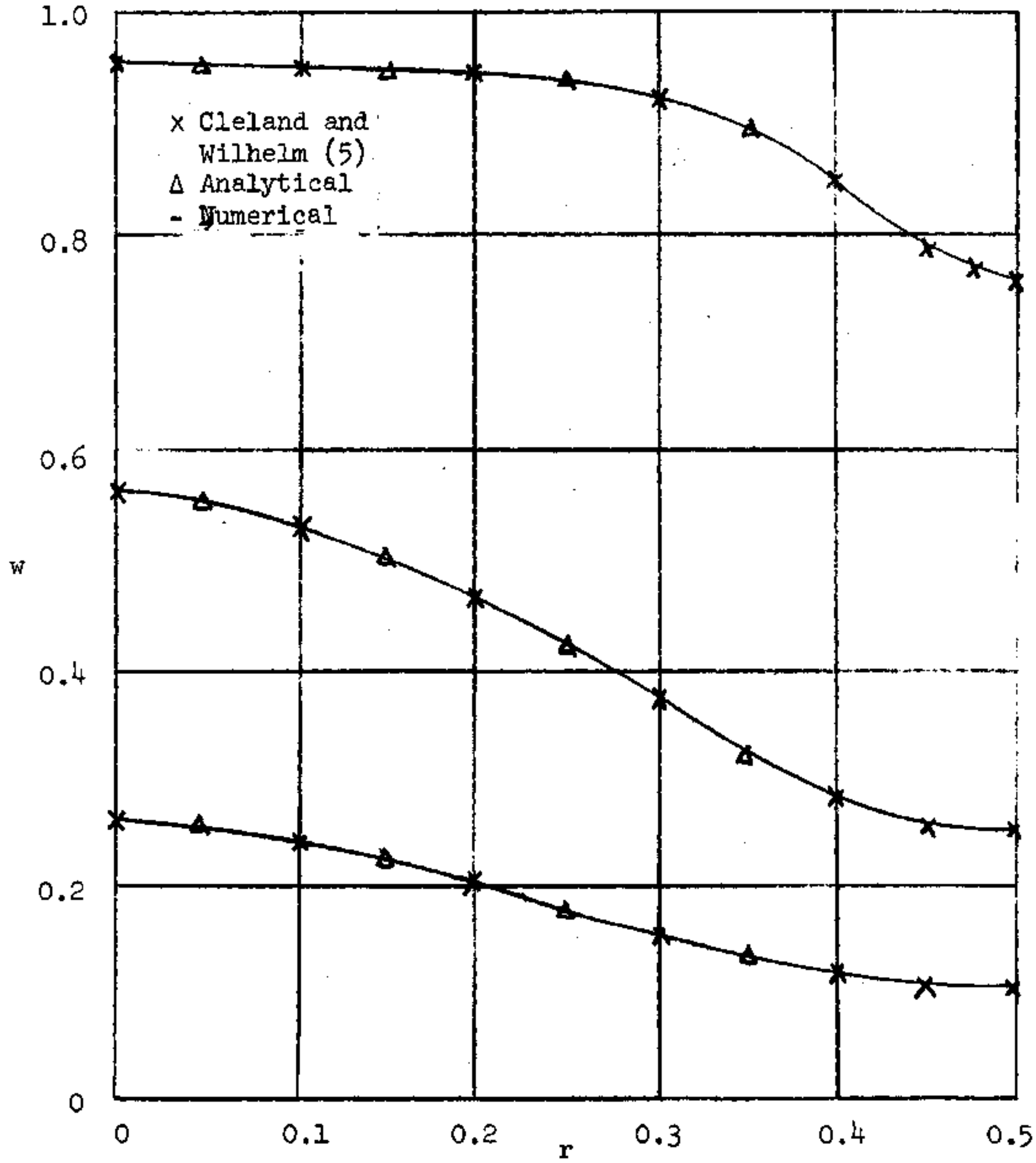


Figure 1. Comparison of Profiles
$\alpha = 2.5$
$n = 1$

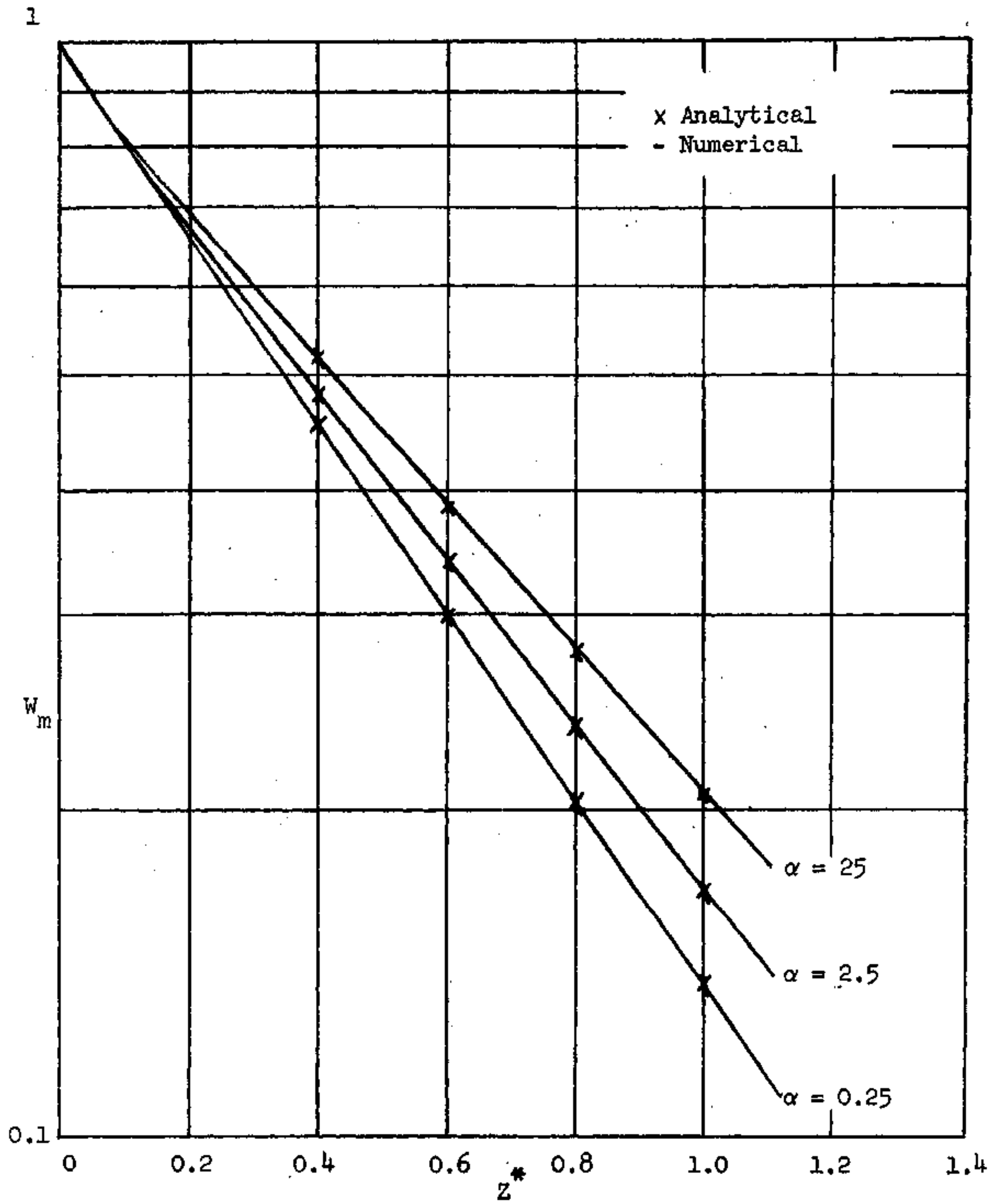


Figure 2. Comparison of Average Concentration Axial Profiles.

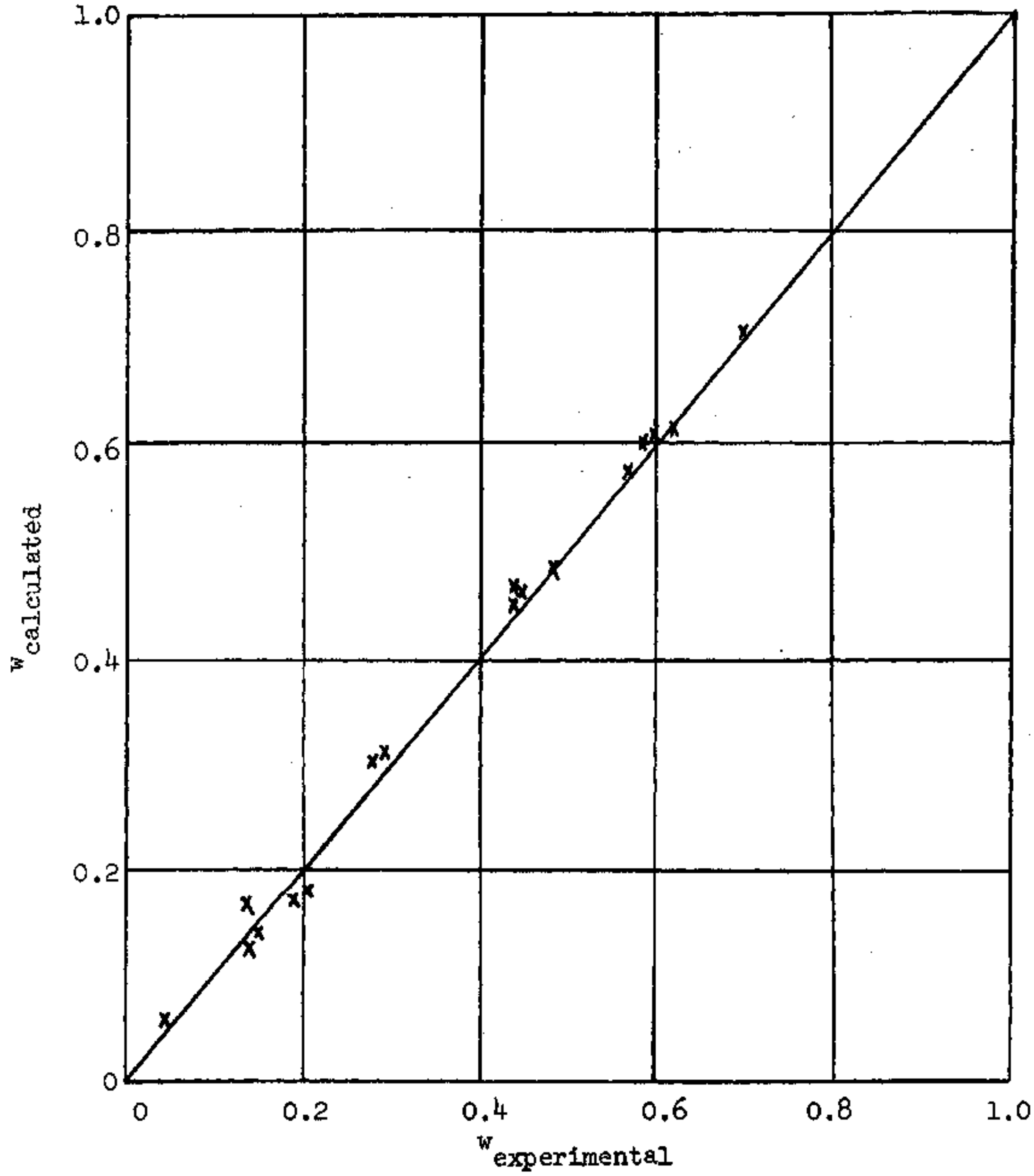


Figure 3. Comparison of Calculated and Experimental Results
Data of Cleland and Wilhelm (5)

Heat and Mass Transfer

Eigenvalues, expansion coefficients and norms are presented for values of $n = 0.2, 0.4, 0.5, 0.6, 0.8, 1.0$ and 1.5. The results are presented in Table 13 of Appendix E. Values of various Nusselt numbers and the mean temperature tabulated as a function of ξ in Table 14 of Appendix E.

Eigenvalues for $n = 1, 0.5$ and 0.2 have been reported by Bird (16) and are in good agreement with those presented here. Comparison of numerical and analytical solutions is given in Figures 5, 6 and 7.

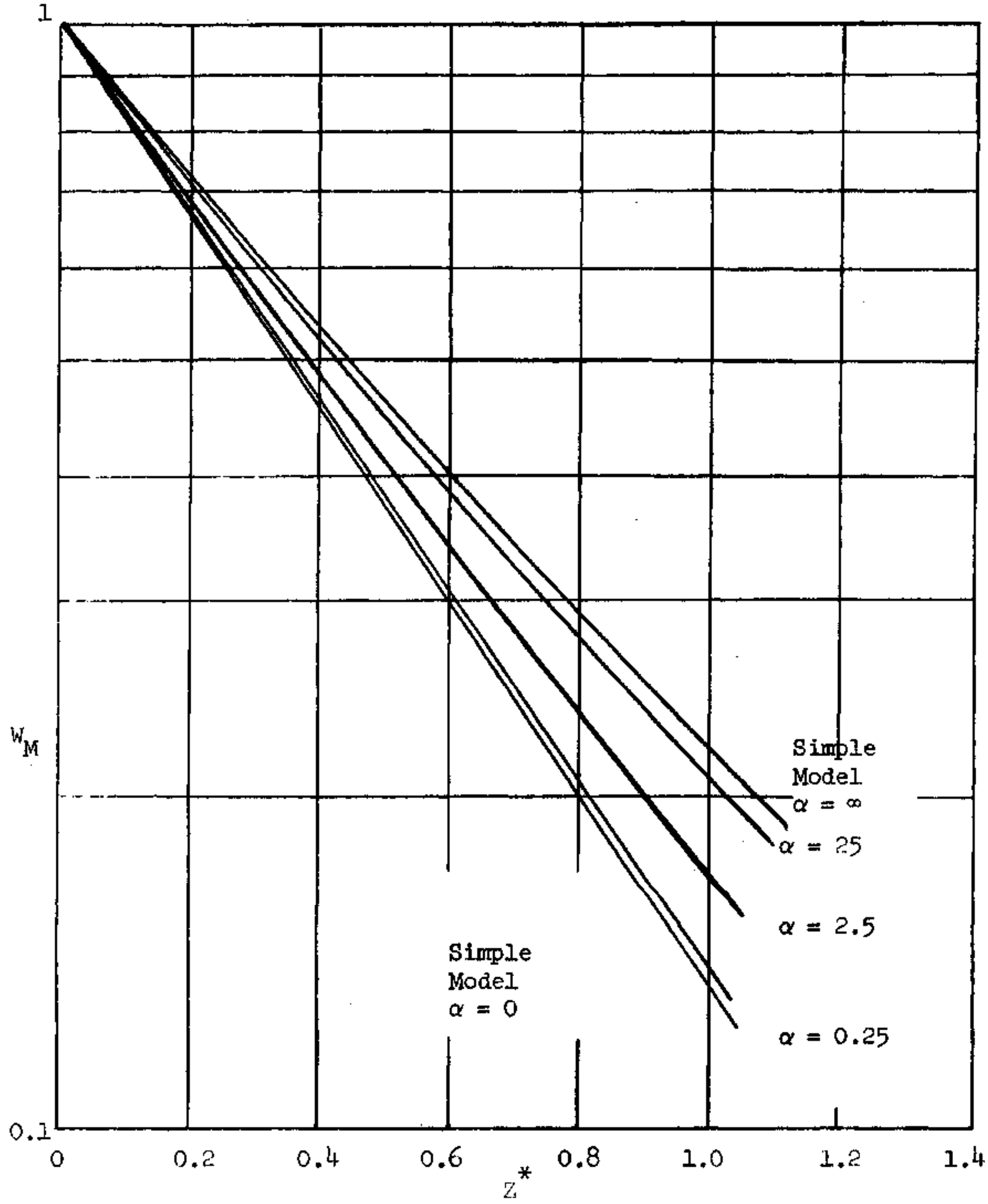


Figure 4. Effect of α for the Tubular Reactor Problem.

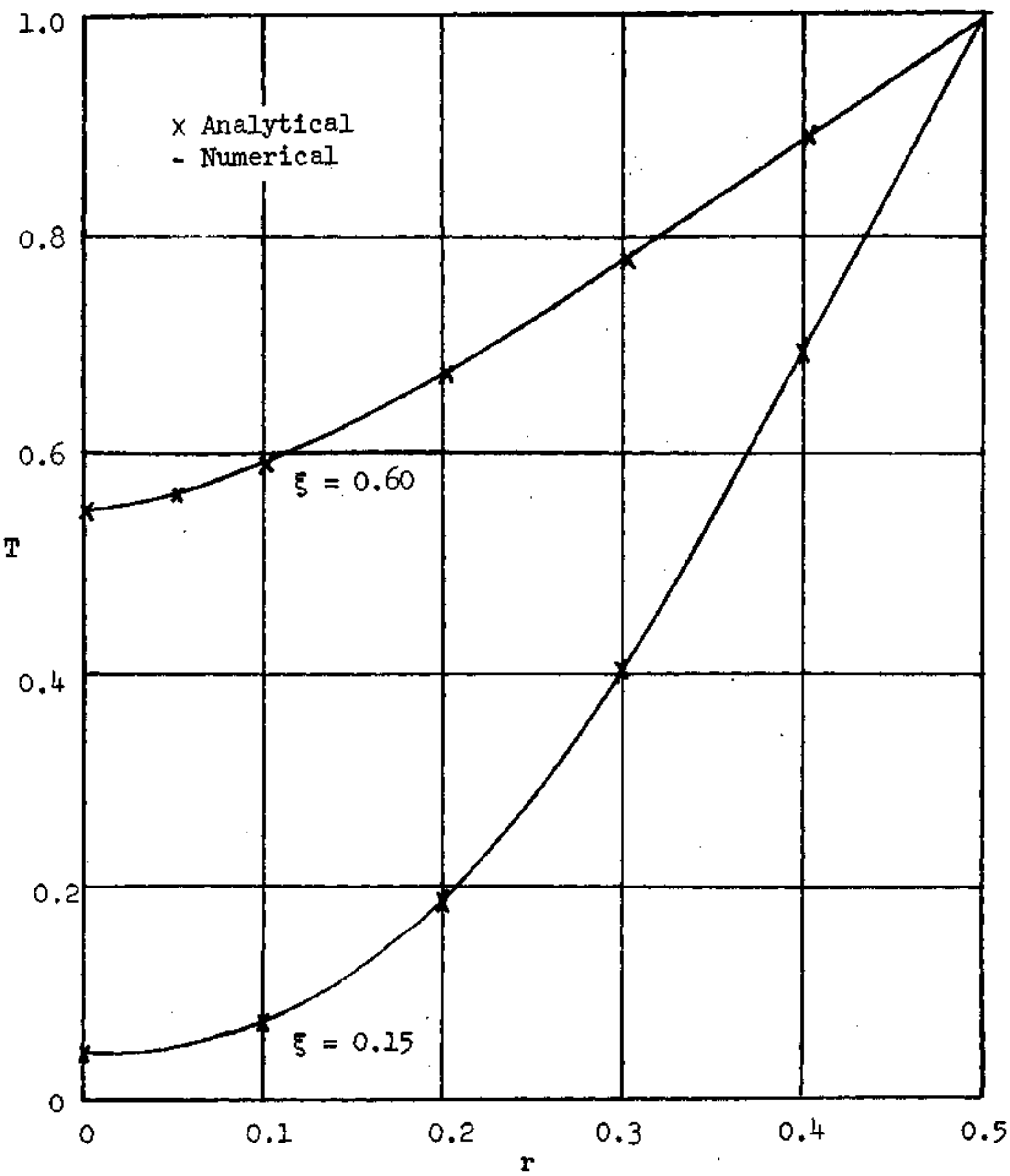


Figure 5. Comparison of Analytical and Numerical Solutions
Radial Temperature Profiles
$n = 1.5$

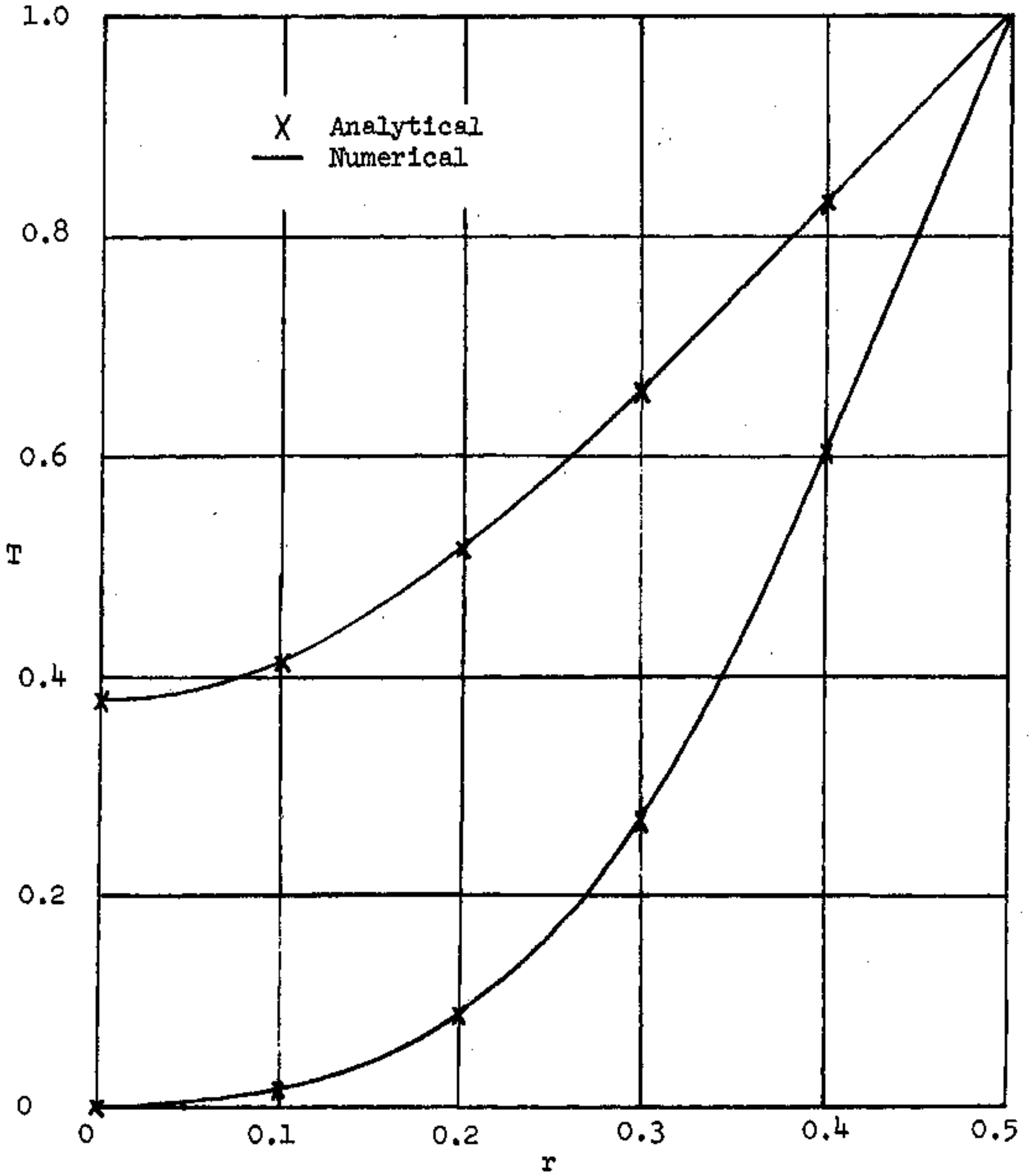


Figure 6. Comparison of Analytical and Numerical Solutions
Radial Temperature Profiles
n = 0.2.

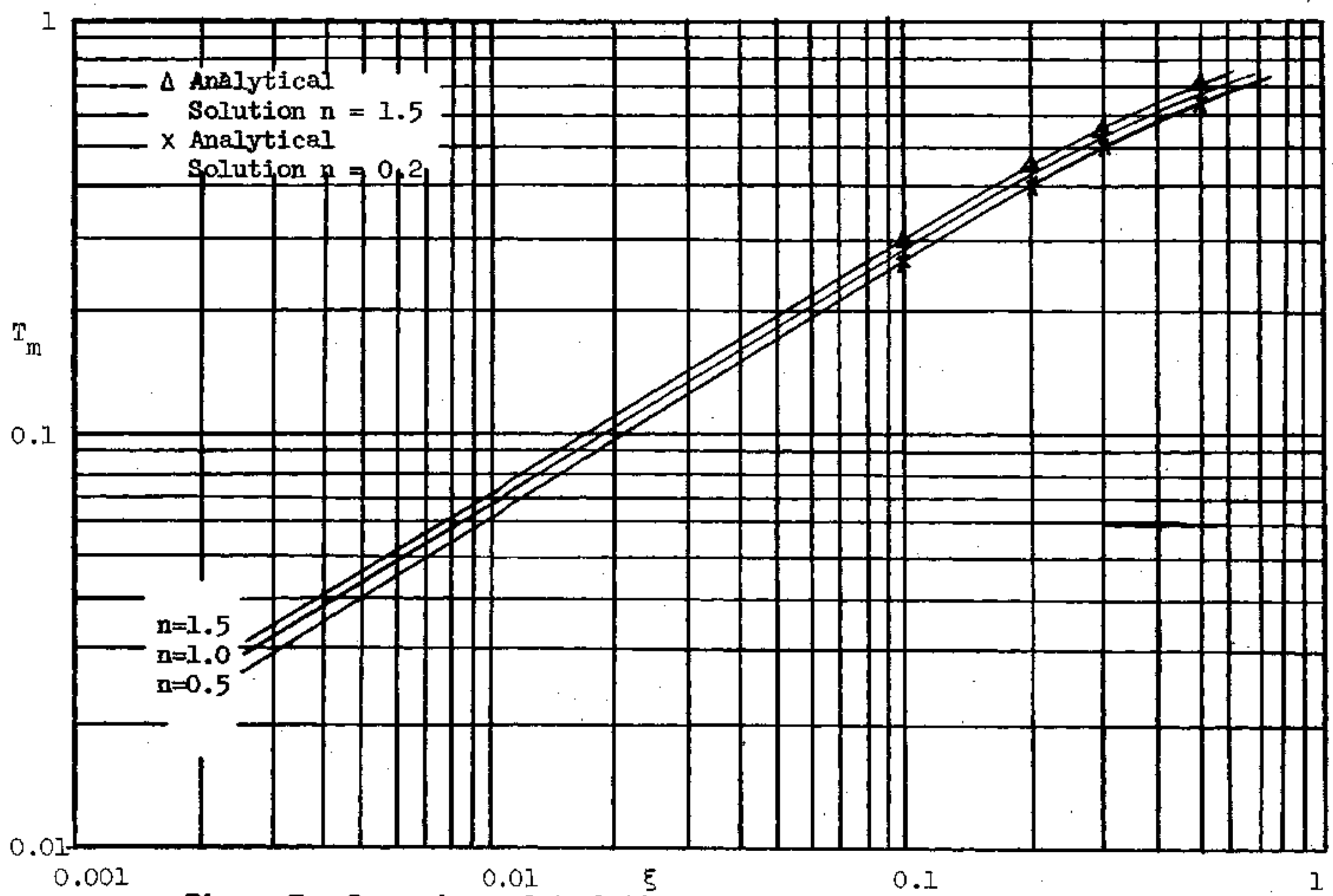


Figure 7. Comparison of Analytical and Numerical Solutions Average Temperature Profiles.

CHAPTER V

NUMERICAL SOLUTIONS

The primary purpose of this study was to obtain solutions to problems involving non-Newtonian fluids. Solutions for several simplified situations were presented in Chapter III and analytical solutions for the constant property, fully developed flow problems were presented in Chapter IV. In this chapter numerical solutions to Equations II-4, II-9, and II-12 which are valid in the entrance region of the tube and which consider the fluid properties to be realistic functions of temperature and concentration are discussed. Details of the solution may be found in Appendix A and the results presented in tabular form may be found in Appendices F and G.

A complete solution to Equations II-4, II-9, and II-12 involves six parameters and eight property variables. These are

Parameters	Properties
α Sc θ	ρ K_v n C_p
Pr Br T_W	k K_r D H

The primary distinction between the parameters and the properties is that the parameters are constant for any given program, while the properties are often functions of temperature and composition. A complete study of all ranges of variables would require an excessive amount of computer time. The results of this study should be considered more as an outline than as an exhaustive analysis.

Validity of Scheme

Before a numerical scheme can be accepted as correct, some criteria must be used to test the validity of the solution obtained. A rigorous stability and convergence analysis of the schemes used in this study would be difficult if not impossible at this time. An analysis procedure for single partial differential equations based on intuitive arguments has been presented by O'Brien (36) and outlined by Hildebrand (37). This procedure is based on the unpublished work of von Neumann. This method has been extended to systems of equations by Lax (38) and Richtmeyer (39). Bodoia (40) has given a thorough adaptation of this method to the solution of the momentum and continuity equations. Wilkins (15) has extended this to include the energy equation. Addition of the diffusion equation presents no new analysis. The conclusion is that the scheme used here is unconditionally stable and convergent.

In addition, the accuracy of the scheme can be tested by comparison with the results of the analytical solutions and experimental data. These comparisons have been presented in Figures 1, 2, 3, 5, 6, and 7. The solution to the momentum equation gives results identical to those of Wilkins (15). The heat transfer results for variable viscosity for Newtonian fluids agree well with those of Wilkins (15).

Further, by the use of various internal checks it was found that the tubular reactor problem is a far more stable problem than the heat-transfer problem. For the heat-transfer problem the heat added to an element of fluid can be determined either by the axial rise in the mean temperature for a given step or by the slope of the temperature profile at the tube wall. The agreement between these two independent methods provides a good check for the accuracy of the solution. For the reactor problem a similar check can be made if account is made of the heat liberated or absorbed by the reaction. These features are discussed in Appendix A. These checks were used to determine the step size used to march down the tube. The time required for a given run was therefore a strong function of the stability of the solution.

In this study it was found that a tubular reactor program could be run on a Burroughs B-5500 computer in about 100 seconds for fully developed flow and about 200 seconds for developing flow. The heat-transfer programs required about 250 seconds for fully developed flow and about 600-800 seconds for developing flow. For heat-transfer programs the first step requires that the temperature at the wall take a step function change. The velocity profile also changes most drastically very close to the wall. To achieve accurate solutions, small steps must be taken. The reactor problem is a homogeneous reaction that occurs all the way across the tube. The violent changes which occur at the wall occupy such a small part of the total volume that larger steps may be taken and still achieve accurate solutions.

In summary the equations of motion and energy can easily be documented to show the validity of the solutions. Wilkins (15) has presented numerous comparisons of his results with experimental data and the scheme used here is similar in many respects to the one used by Wilkins (15). The solution to the diffusion equation has been compared to the little experimental data available in Figure 3. Therefore, it is felt that the solutions presented here are stable, convergent and accurate and do in fact represent the physical situation.

Tubular Reactor

Constant Properties - Fully Developed Flow

For constant properties and fully developed flow the only parameter of interest is α. The two simple models representing limiting values as α approaches zero and infinity are given in Table 4 of Appendix D. The numerical solution presented in Tables 15 and 16 of Appendix F provides a convenient means of interpolating between the two limiting values. This effect has been illustrated in Figure 4. It is seen that diffusion is important for $\alpha \leq 25$. Notice that for large values of α the average concentration profile is independent of α but that the radial concentration profile continues to change, and the wall concentration approaches zero.

At this point it is worthwhile to consider the range of interest of the parameters. It is assumed in this work that many reactions of interest will be at least 90 per cent complete at a tube length of between 10 and 500 diameters. Considering the extreme values of the Reynolds number to be 100 and 2200 gives

$$0.005 \leq z \leq 5$$

Using the "plug flow" and "parabolic flow" models, the values of α/Sc are determined to be approximately

$$0.05 \leq \frac{\alpha}{Sc} \leq 25$$

Since gases normally have Schmidt numbers of about one, it is seen that diffusion effects for gases can be significant. Liquids normally have much higher Schmidt numbers. Therefore, even for low values of α/Sc,

α is relatively high and diffusion effects are small. For liquids the "parabolic flow" model results presented in Table 4 of Appendix D represent a good correlation. The effect of the flow consistency index is shown in Figure 8.

Constant Properties - Developing Flow

If the entering velocity profile is considered to be uniform, then the velocity profile changes as the fluid moves down the tube. The results of many investigators indicate that the length required for the centerline velocity to reach 99 per cent of its final value is about

$$z = 0.060$$

for Newtonian fluids. Since this process occurs asymtotically most of the effect of the developing velocity profile on the concentration profile has occurred in a much shorter distance. From these remarks it is expected that developing flow will affect the result only if

$$\frac{\alpha}{Sc} \geq 1$$

For non-Newtonian fluids the hydrodynamic entrance length is longer than 0.060 for values of n less than one and shorter than 0.060 for values of n greater than one. Entrance lengths results for non-Newtonian fluids are presented below.

n	Entrance length, z (99% Criteria)
1.5	0.022
1.0	0.060
0.5	0.11
0.2	0.16

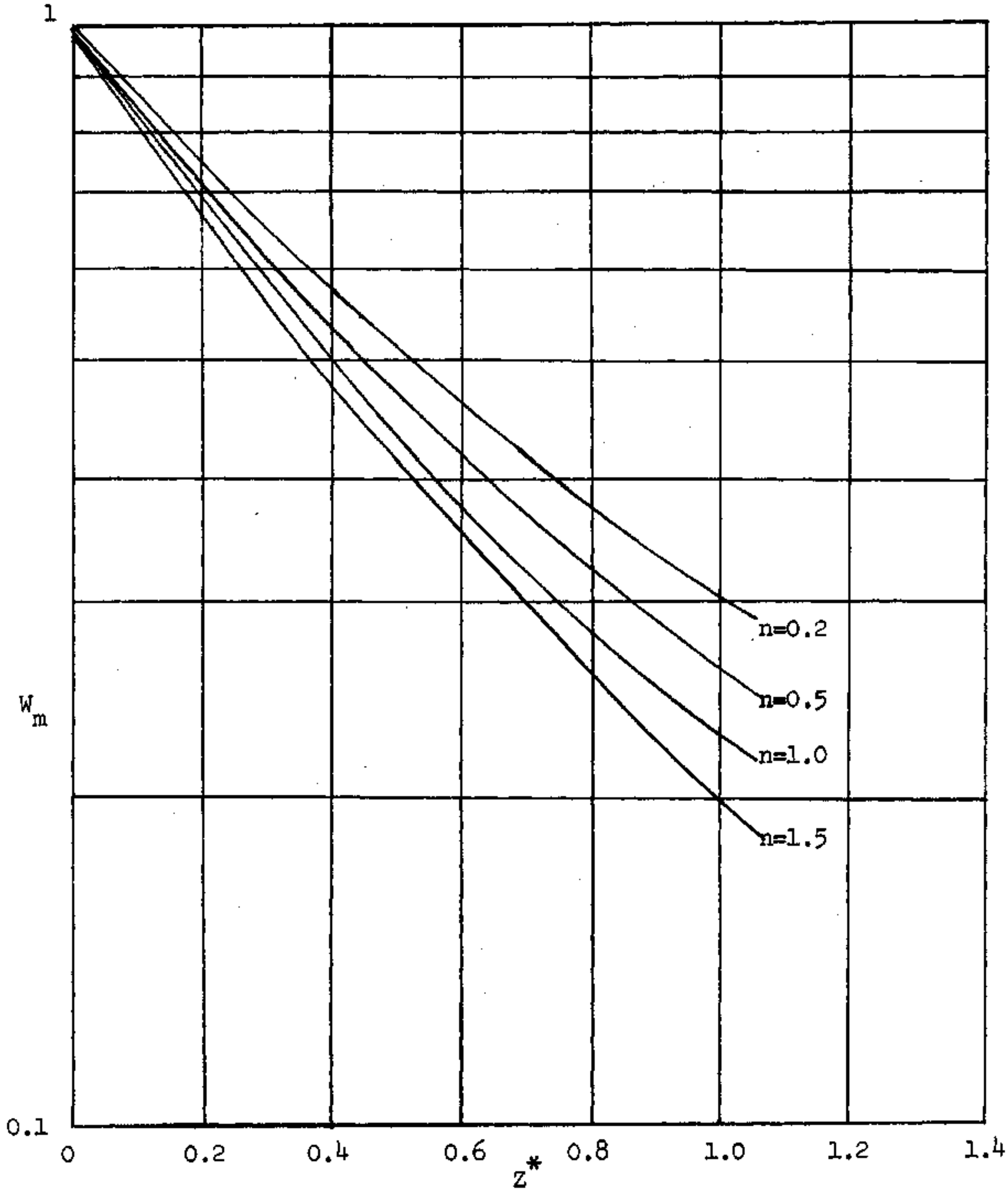


Figure 8. Effect of Flow Consistency Index on Conversion Parabolic Flow Model.

These entrance lengths agree well with those of Collins (34). Developing axial velocity profiles for non-Newtonian fluids are presented in Table 1 and illustrated in Figure 9.

The effect of developing axial velocity profiles on the concentration profiles is given in Tables 17 and 18 of Appendix F and is illustrated in Figures 10, 11 and 12. The "plug flow" model represents the lower bound for the developing flow problem. Therefore, for liquids the curve will usually rest on the "parabolic flow" model but will move towards the "plug flow" model for high values of α/Sc. The limiting cases are presented in Figure 13. Although a larger difference is possible for values of n greater than one, the entrance length is so short that no effect is felt except for very high values of α/Sc. Although the solutions are functions of α/Sc and Sc, the effect of Sc can be taken into account by using separate tables for gases and liquids.

Most of the results reported in this work are for a parabolic entrance velocity profile, and usually these results will be adequate. For non-Newtonian fluids, developing flow will affect the results for

$$\frac{\alpha}{Sc} \geq n$$

For reactions which occur substantially in the entrance region a correction must be made towards the "plug flow" model

Variable Density

If the density is allowed to vary with concentration, then the parameters of the solution are α, Sc and θ. A change in density produces a change in the residence time in the tube. This effect has been determined by the simple models of Chapter III. In addition, the con-

Table 1. Developing Velocity Profiles for Non-Newtonian Fluids

		u	u	u	u	u
n	z x 10^2	r = 0	r = 0.2	r = 0.3	r = 0.4	r = 4.5
1.5	0.001	1.028	1.028	1.028	1.028	1.025
	0.028	1.226	1.226	1.226	1.114	0.685
	0.092	1.360	1.360	1.348	1.948	0.522
	0.508	1.739	1.642	1.320	0.754	0.396
	0.918	1.958	1.688	1.288	0.714	0.373
	1.327	2.081	1.705	1.273	0.698	0.363
	2.249	2.168	1.719	1.261	0.645	0.356
	3.478	2.195	1.725	1.263	0.685	0.355
0.5	0.382	1.157	1.150	1.142	1.078	0.856
	1.124	1.302	1.292	1.249	1.021	0.654
	2.455	1.438	1.405	1.292	0.930	0.550
	4.503	1.543	1.481	1.308	0.874	0.499
	6.961	1.600	1.520	1.310	0.844	0.475
	11.467	1.642	1.548	1.309	0.824	0.461
	24.574	1.669	1.560	1.308	0.815	0.453
0.2	1.532	1.131	1.131	1.124	1.070	0.887
	4.092	1.221	1.221	1.197	1.046	0.767
	7.369	1.274	1.271	1.235	1.025	0.694
	16.380	1.319	1.315	1.264	0.994	0.639
	26.211	1.331	1.326	1.270	0.985	0.627

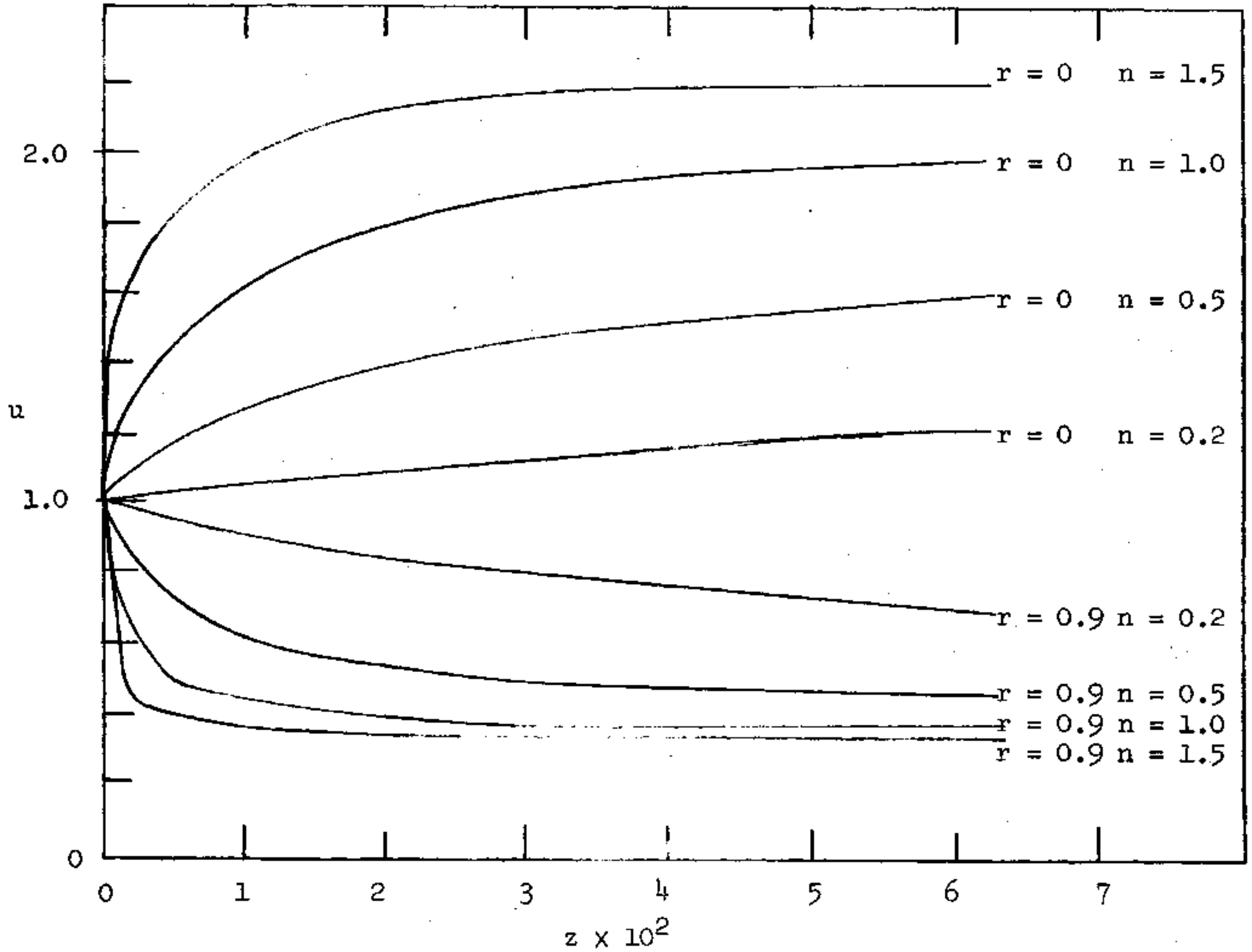


Figure 9. Axial Velocity Profiles for Non-Newtonian Fluids.

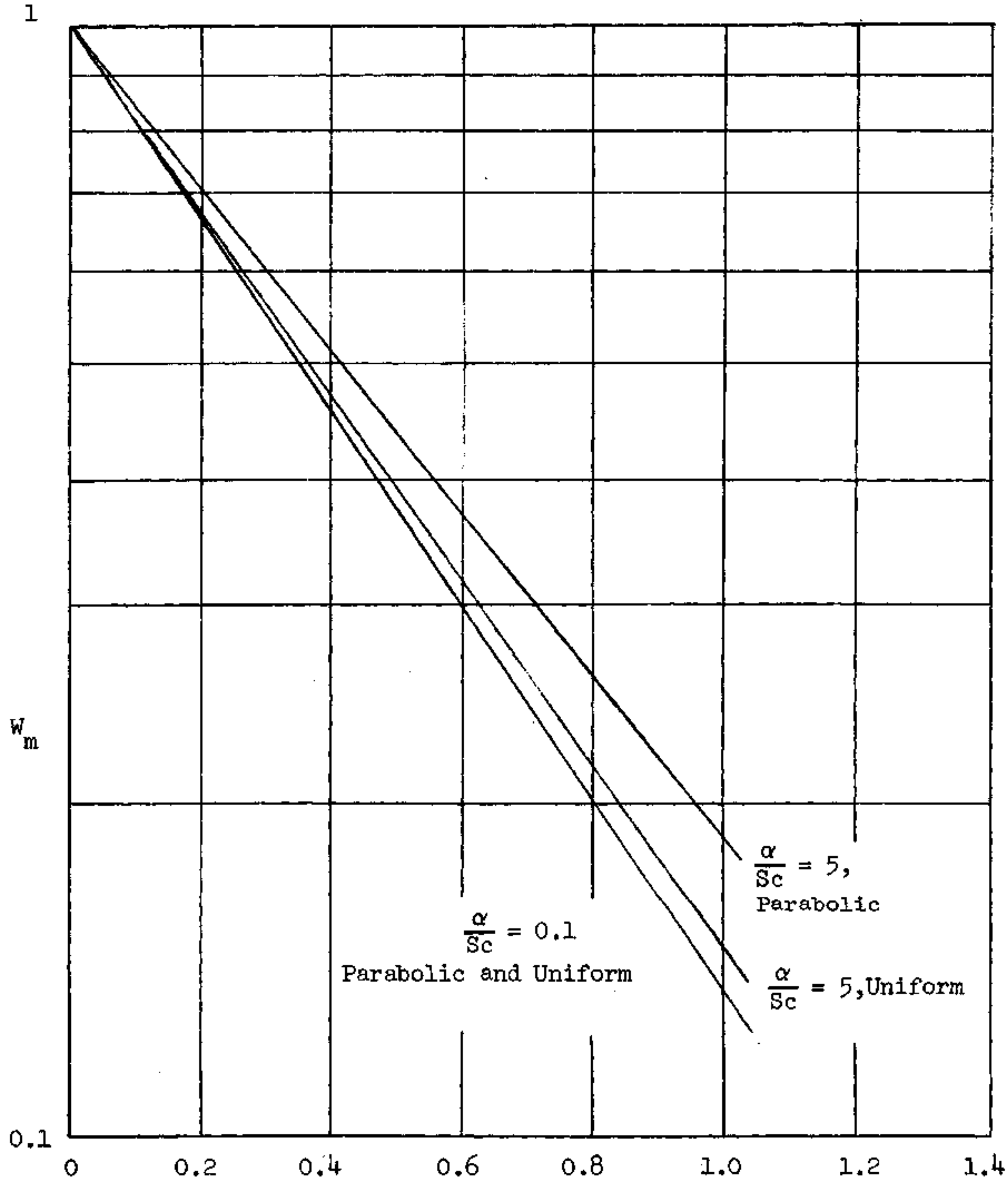


Figure 10. Effect of Entrance Velocity Profile on Average Concentration -- Gases.

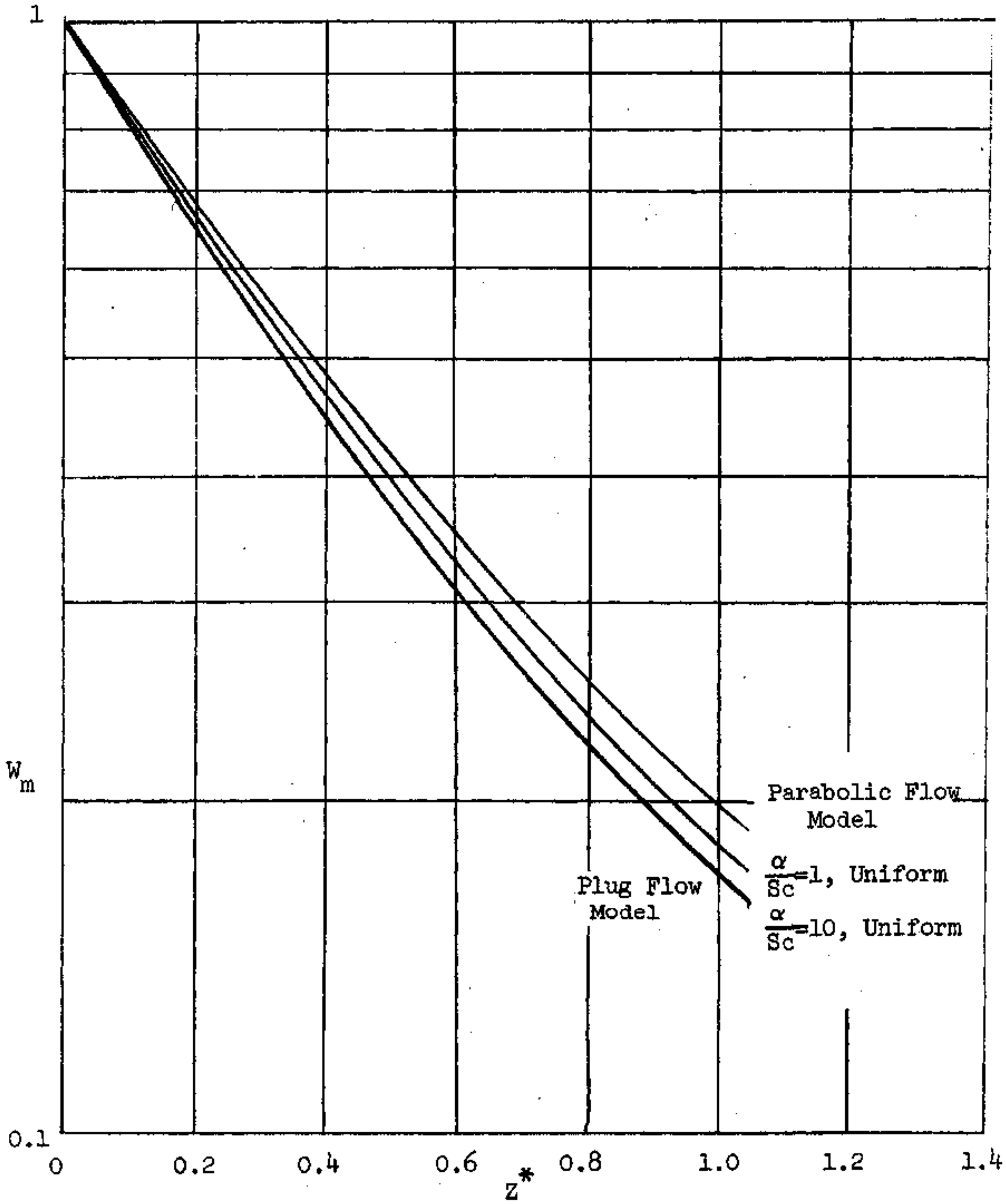


Figure 11. Effect of Entrance Velocity Profile on Average Concentration
Non-Newtonian Fluids
n = 1.5

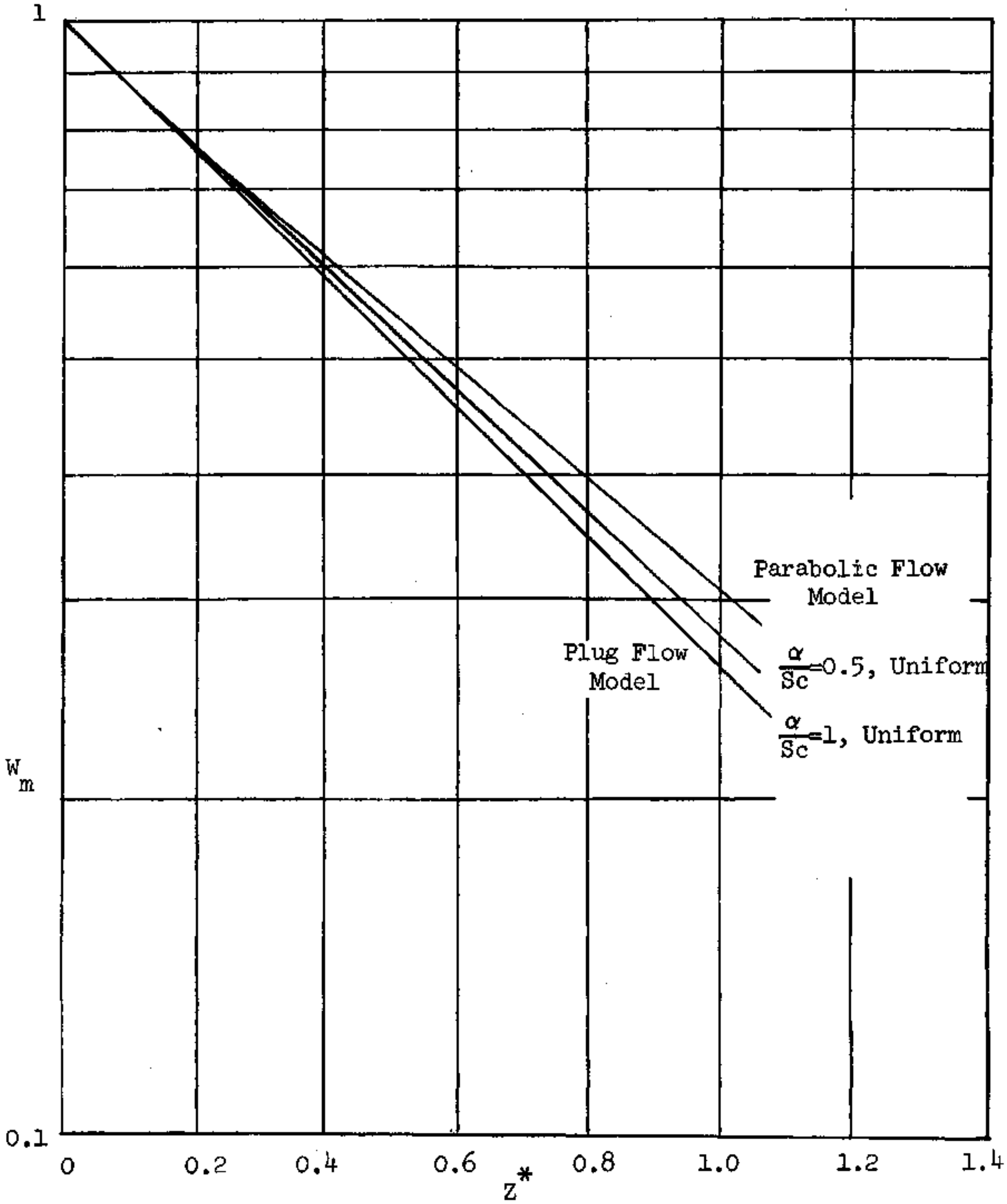


Figure 12. Effect of Entrance Velocity Profile on Average Concentration
non-Newtonian Fluids
n = 0.2.

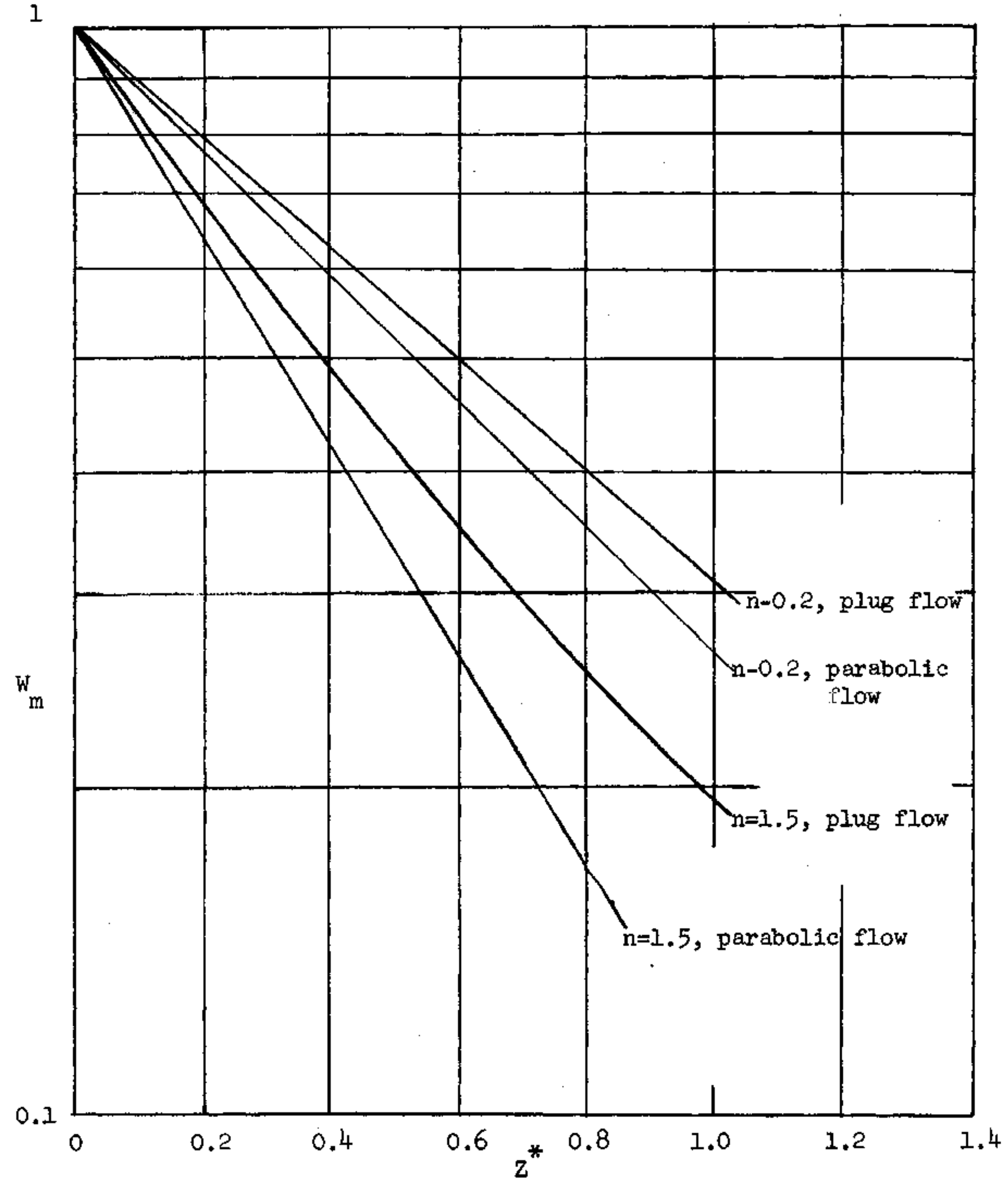


Figure 13. Limiting Effects of Entrance Velocity Profile

centration profile across the tube establishes free- convective forces which also affect the yield.

The results for gases may be correlated by the simple models. Interpolation between the two curves is aided by the information in Table 19 of Appendix F and illustrated in Figures 14 and 15. For low values of α/Sc, free-convection effects are negligible and the results are independent of Sc. For high values of α/Sc, free-convection effects, although not highly significant, are noticeable. These effects are correlated by a parameter, Fc where

$$Fc = \left(\frac{\rho_{out}}{\rho_o}\right)^2 \left(\frac{\rho_{out} - \rho_o}{\rho_{out} + \rho_o}\right) \theta \qquad \text{V-1}$$

These results are given in Table 20 of Appendix F.

The results for liquids may be correlated to within a few per cent by the "parabolic flow" model. Since Sc is large for liquids, even small values of α/Sc correspond to large values of α.

Reactions with Heat Effects

Since the rate of a chemical reaction is a strong function of temperâture, the processes controlling heat transfer will also determine the yield of the reaction. The simple model approach suggests that the heat transfer with the environment may be characterized by the group $\alpha Pr/Sc$. For low values of this group, heat transfer is large and the solution tends towards that of isothermal flow. For high values of $\alpha Pr/Sc$, heat transfer is low and the solution tends toward the adiabatic flow solutions. Indication of this group as a correlating parameter is given in Table 2. It was found that separate correlations for gases and liquids was advisable since the factors comprising the correlating group

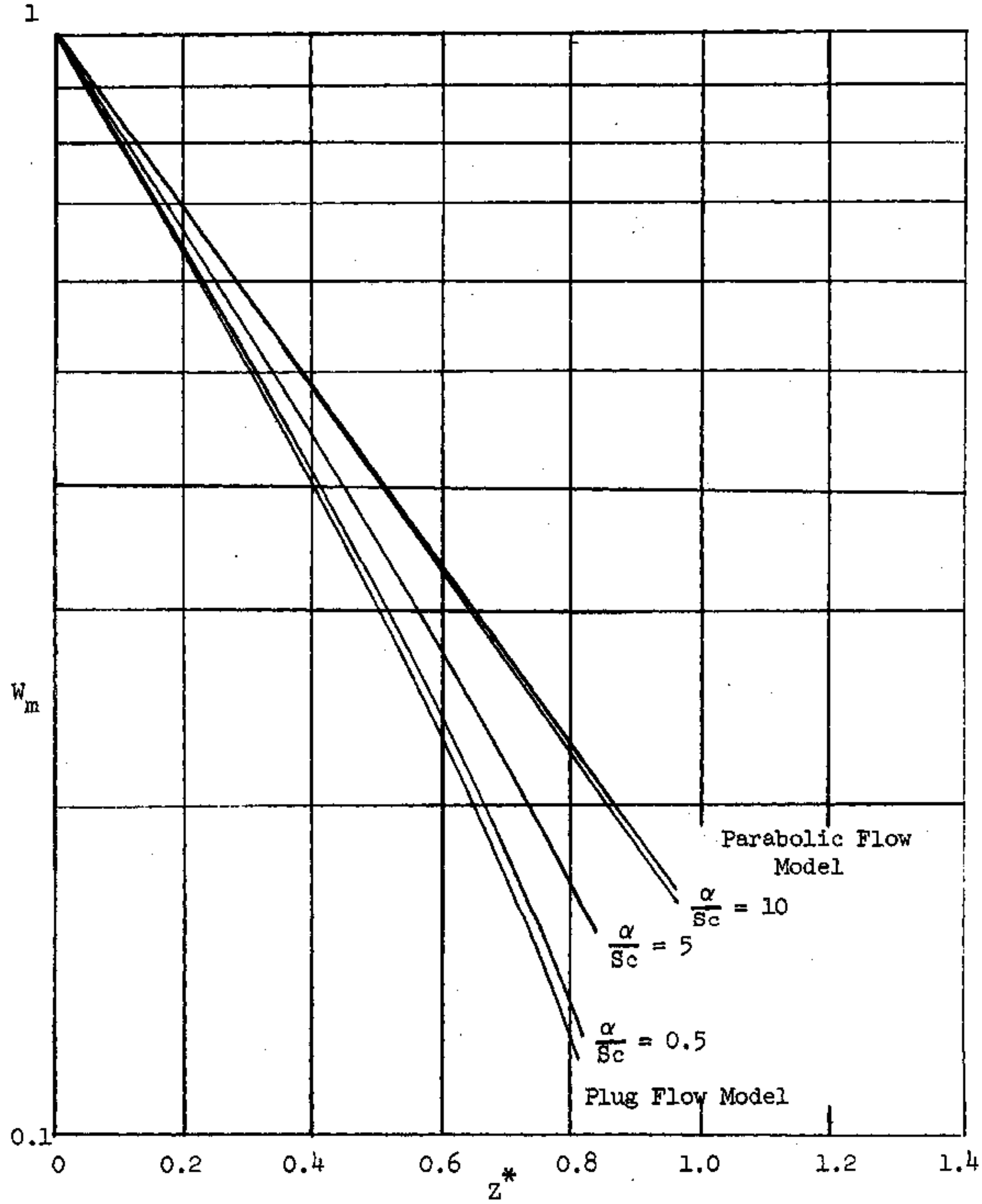


Figure 14. Effect of $\frac{\alpha}{Sc}$ for Variable Density

$$\frac{\rho_{out}}{\rho_o} = 1.5$$

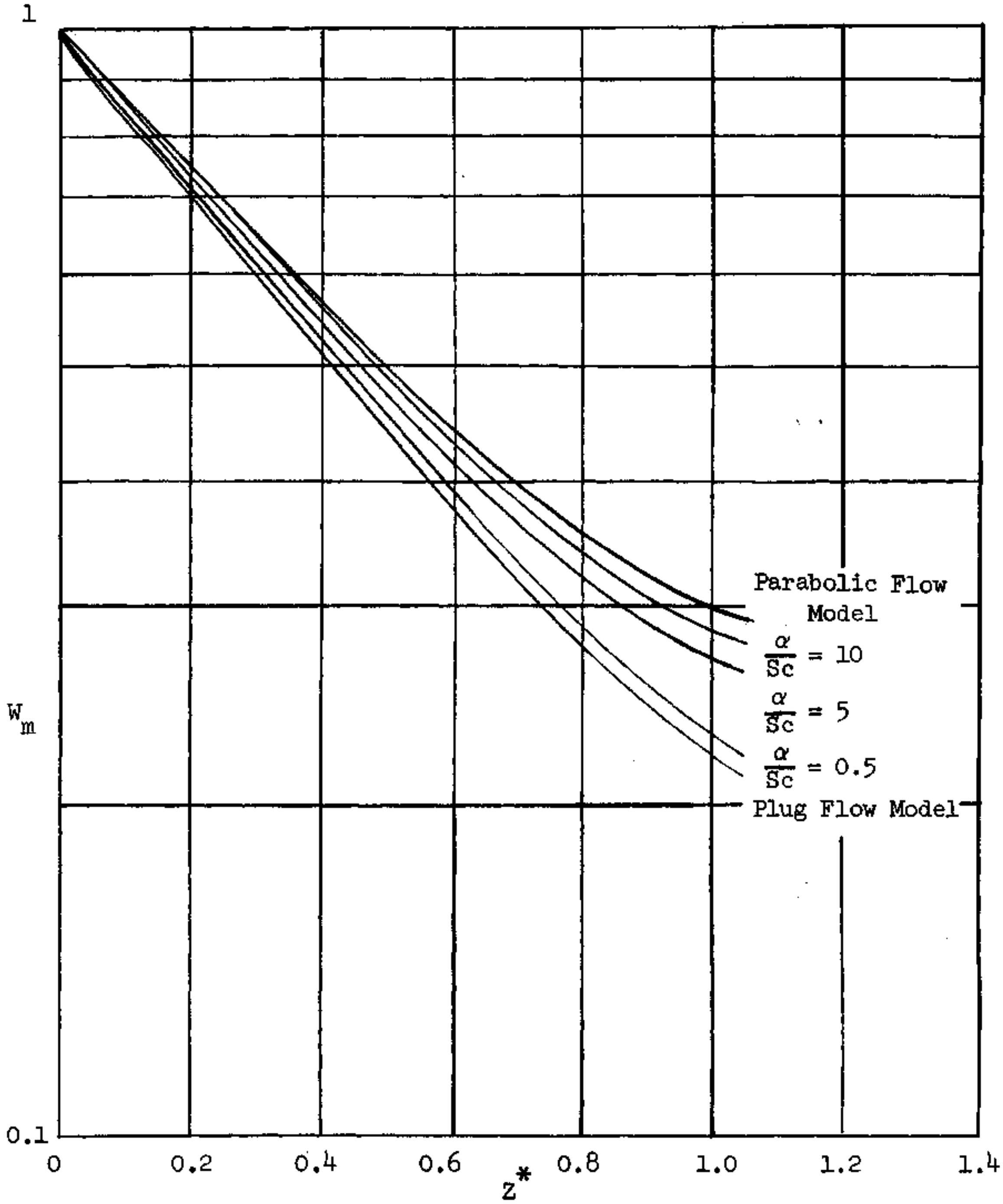


Figure 15. Effect of $\frac{\alpha}{Sc}$ for Variable Density

$$\frac{\rho_{out}}{\rho_o} = 0.5$$

Table 2. Indication of $\alpha Pr/Sc$ as Correlating Parameter

n	α	Pr	Sc	$\frac{\alpha Pr}{Sc}$	$\overset{\blacktriangle}{\Delta H}$	E_a	W	T	Z^*	r = 0		r = 0.2		r = 0.4	r = 0.5	
0.5	1,000	100	10,000	10	-0.3	21	0.794	1.056	0.079	0.900	1.030	0.891	1.033	0.670	1.106	0.239
							0.596	1.110	0.115	0.830	1.051	0.811	1.057	0.475	1.264	0.076
							0.397	1.165	0.150	0.730	1.081	0.688	1.095	0.092	1.257	0.022
							0.198	1.220	0.187	0.558	1.133	0.445	1.170	0.000	1.243	0.000
							0.086	1.251	0.209	0.371	1.189	0.162	1.259	0.000	1.235	0.000
0.5	10,000	10	10,000	10	-0.3	21	0.800	1.054	0.077	0.902	1.029	0.893	1.032	0.684	1.102	0.243
							0.601	1.109	0.111	0.832	1.051	0.813	1.057	0.055	1.263	0.078
							0.394	1.166	0.151	0.728	1.082	0.686	1.096	0.000	1.257	0.000
							0.200	1.220	0.187	0.558	1.133	0.447	1.170	0.000	1.243	0.000
							0.090	1.250	0.209	0.377	1.188	0.171	1.256	0.000	1.235	0.000
0.2	25	10	1,000	0.25	-0.1	14	0.795	1.010	0.167	0.830	1.015	0.830	1.013	0.776	1.006	0.569
							0.595	1.012	0.369	0.640	1.020	0.645	1.017	0.552	1.007	0.339
							0.397	1.010	0.654	0.442	1.017	0.451	1.014	0.340	1.005	0.184
							0.197	1.005	1.185	0.235	1.008	0.242	1.007	0.146	1.003	0.069
							0.088	1.002	1.833	0.115	1.004	0.117	1.003	0.056	1.001	0.024
0.2	250	10	10,000	0.25	-0.1	14	0.796	1.009	0.167	0.830	1.015	0.830	1.013	0.785	1.006	0.485
							0.599	1.012	0.369	0.640	1.020	0.646	1.016	0.578	1.007	0.210
							0.399	1.009	0.664	0.437	1.016	0.448	1.013	0.369	1.005	0.068
							0.193	1.004	1.234	0.222	1.008	0.231	1.006	0.157	1.002	0.010
							0.089	1.002	1.883	0.109	1.003	0.114	1.003	0.060	1.001	0.000
0.2	25	100	10,000	0.25	-0.1	14	0.798	1.010	0.165	0.831	1.015	0.832	1.013	0.778	1.006	0.571
							0.601	1.012	0.362	0.646	1.020	0.651	1.017	0.558	1.007	0.345
							0.398	1.010	0.653	0.442	1.017	0.451	1.014	0.340	1.005	0.180
							0.196	1.005	1.188	0.234	1.008	0.241	1.007	0.145	1.003	0.068
							0.086	1.002	1.848	0.114	1.004	0.115	1.003	0.054	1.001	0.023

vary so widely from gases to liquids. Also the density is a much stronger function of temperature for gases than for liquids.

The parameters of interest then are $\alpha Pr/Sc$, $\Delta\hat{H}$, and the rate E_a at which the reaction rate constant changes with temperature. A number of cases of interest are presented in Appendix F in tabular form. Although this study was intended to be a study of non-Newtonian fluids, it was found that no work has been published for gases or Newtonian liquids. Therefore, results are also presented for these important cases. Use of these results is discussed in Appendix H.

The results may be plotted as the average concentration versus Z^*. It is important to remember that Z^* involves both α/Sc and n so that part of the correlation rests in the choice of the dimensionless distance. A typical graph of W_m versus Z^* for various values of $\alpha Pr/Sc$ is given as Figure 16. Notice that as $\alpha Pr/Sc$ increases, the solution moves from the isothermal line to the adiabatic line. For gases, all ranges of $\alpha Pr/Sc$ are of interest. However, for liquids, if very long tubes are excluded, the results of all values of $\alpha Pr/Sc$ may be summarized by a single line for each value of $\Delta\hat{H}$ and E_a. This convenient liquid correlation, accurate to within about 10%, appears as Figures 17, 18 and 19.

An upper bound of the temperature is that given by adiabatic flow

$$T = 1 - \Delta\hat{H}\,(1 - W) \qquad \text{V-2}$$

One point of interest is that for high values of $\alpha Pr/Sc$ and low degrees of conversion, the maximum temperature occurs near the wall. As the degree of conversion increases, conduction occurs and the maximum

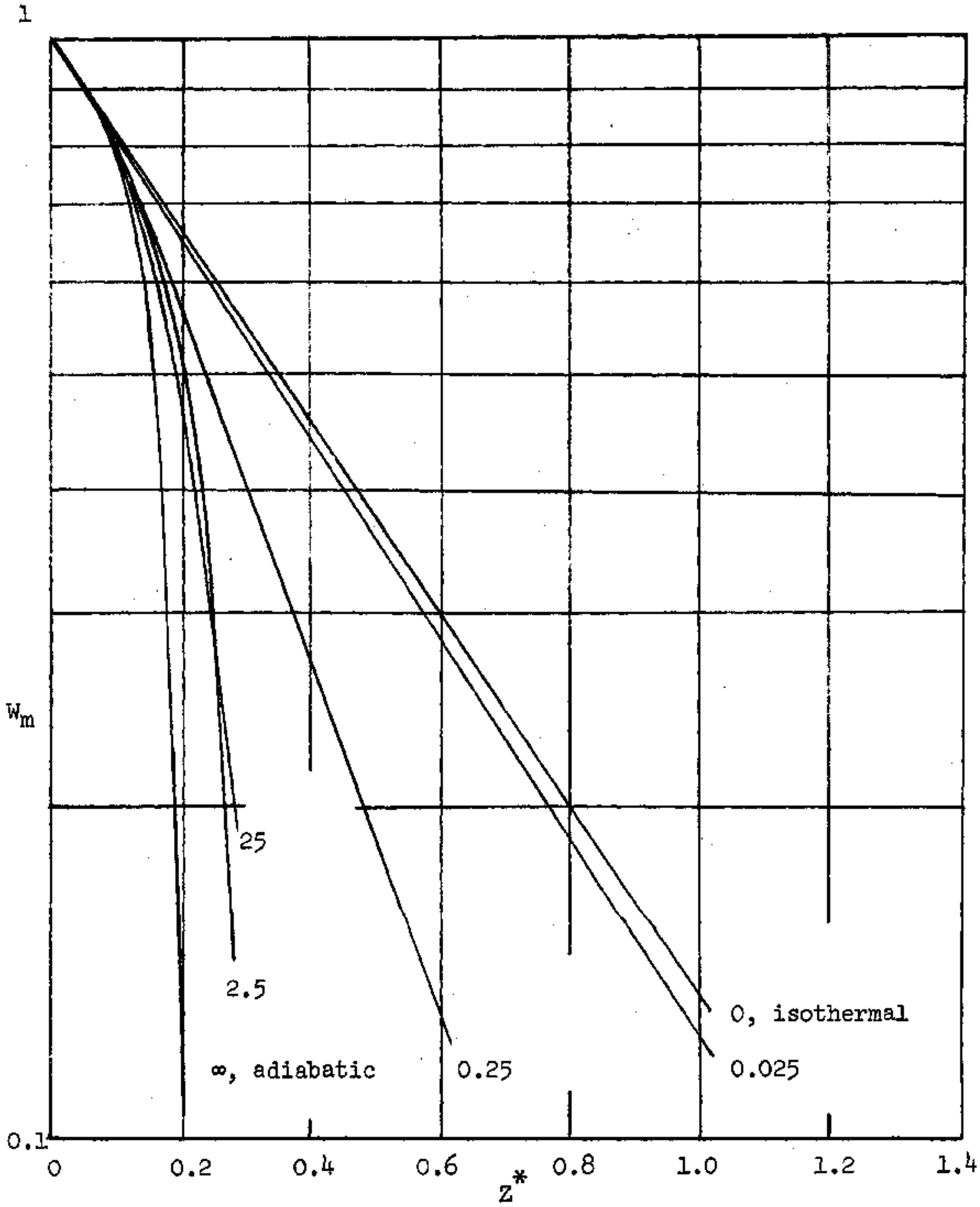


Figure 16. Average Concentration Versus Z^*
Entrance Velocity Profile: Parabolic
Gas
$\Delta\hat{H} = -0.3 \quad E_a = 14$
Parameter: $\frac{\alpha Pr}{Sc}$

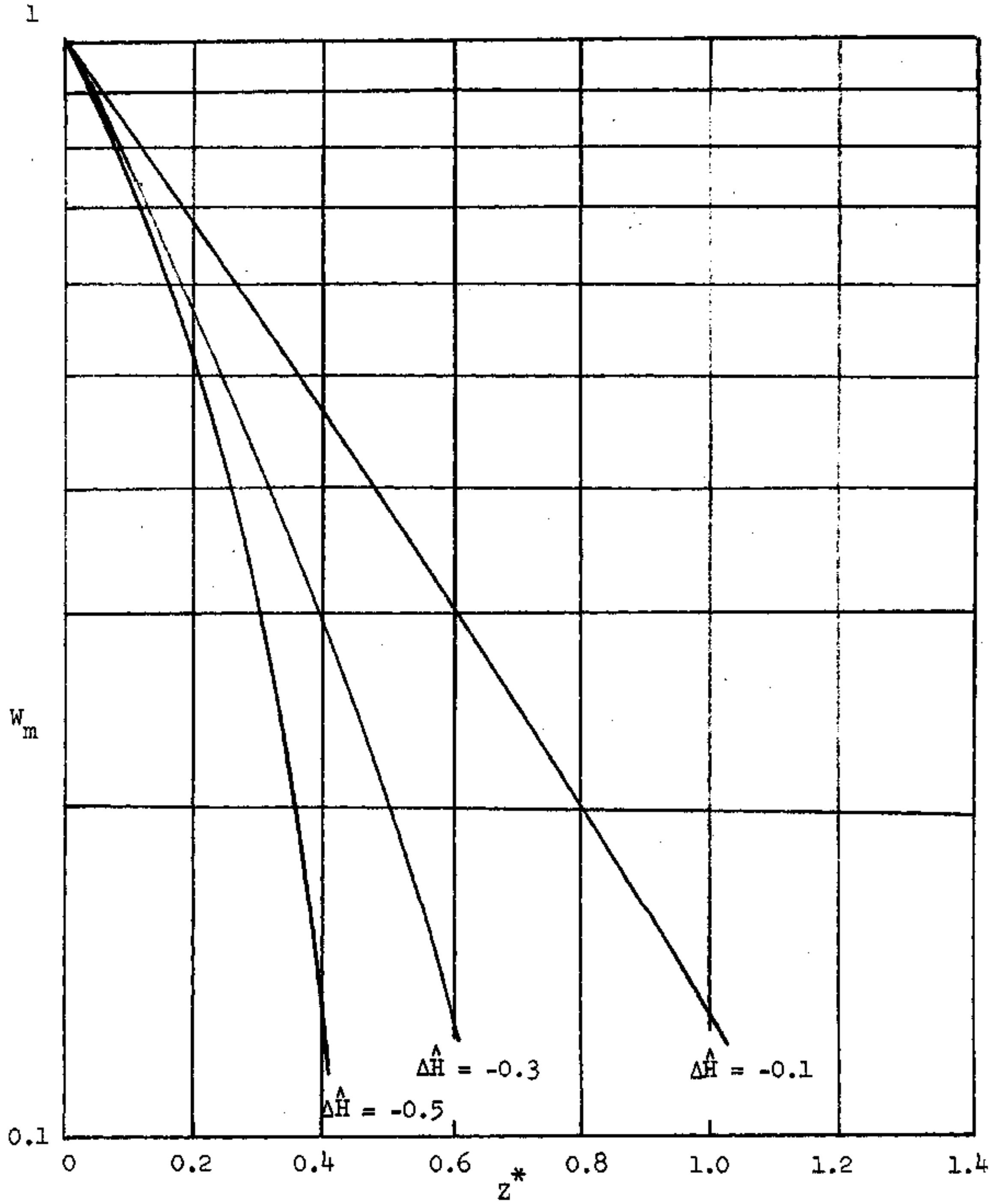


Figure 17. Liquid Correlation
$\alpha Pr/Sc \geq 2.5$
$E_v = 7$

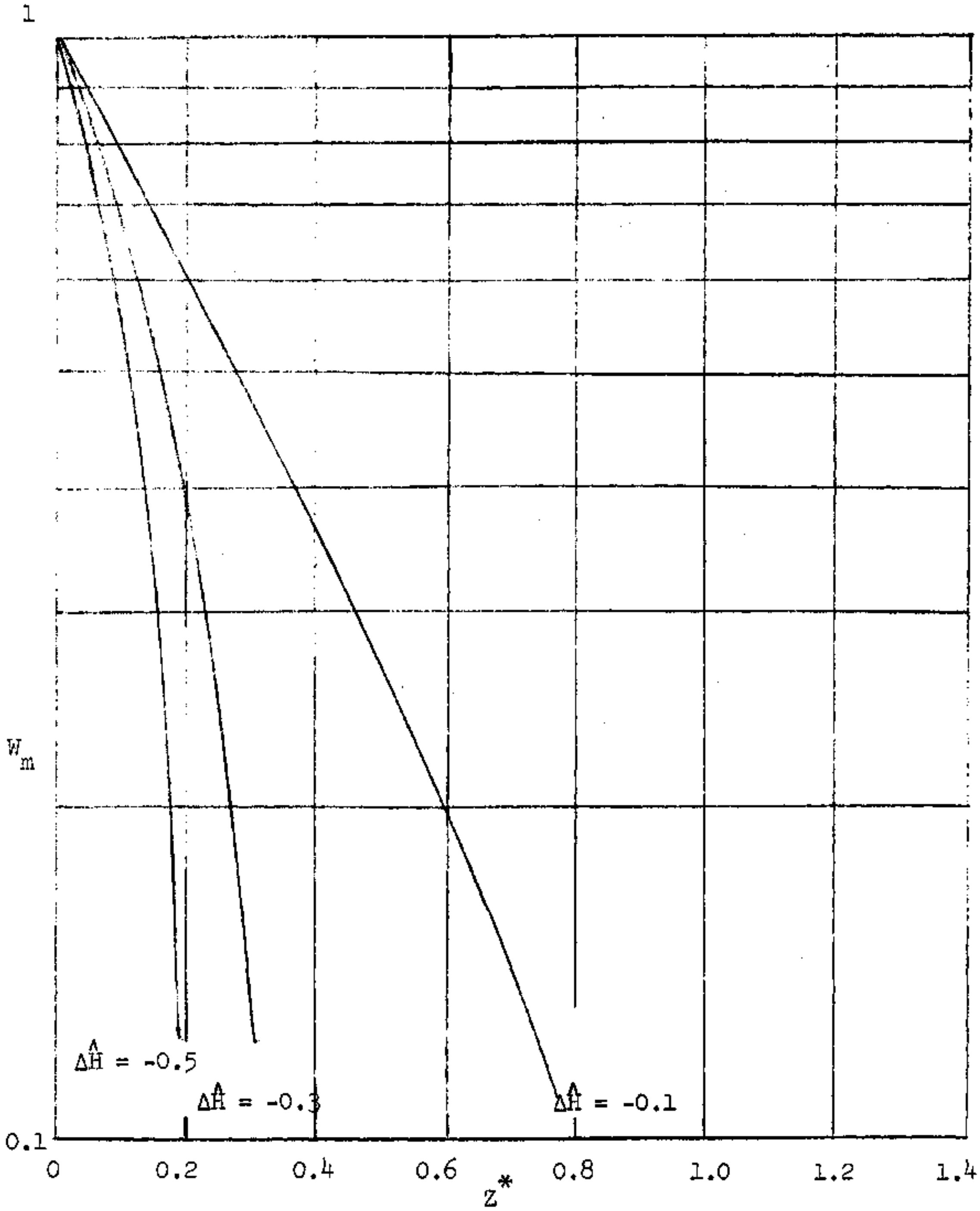


Figure 18. Liquid Correlation
$\alpha Pr/Sc \geq 2.5$
$E_V = 14$

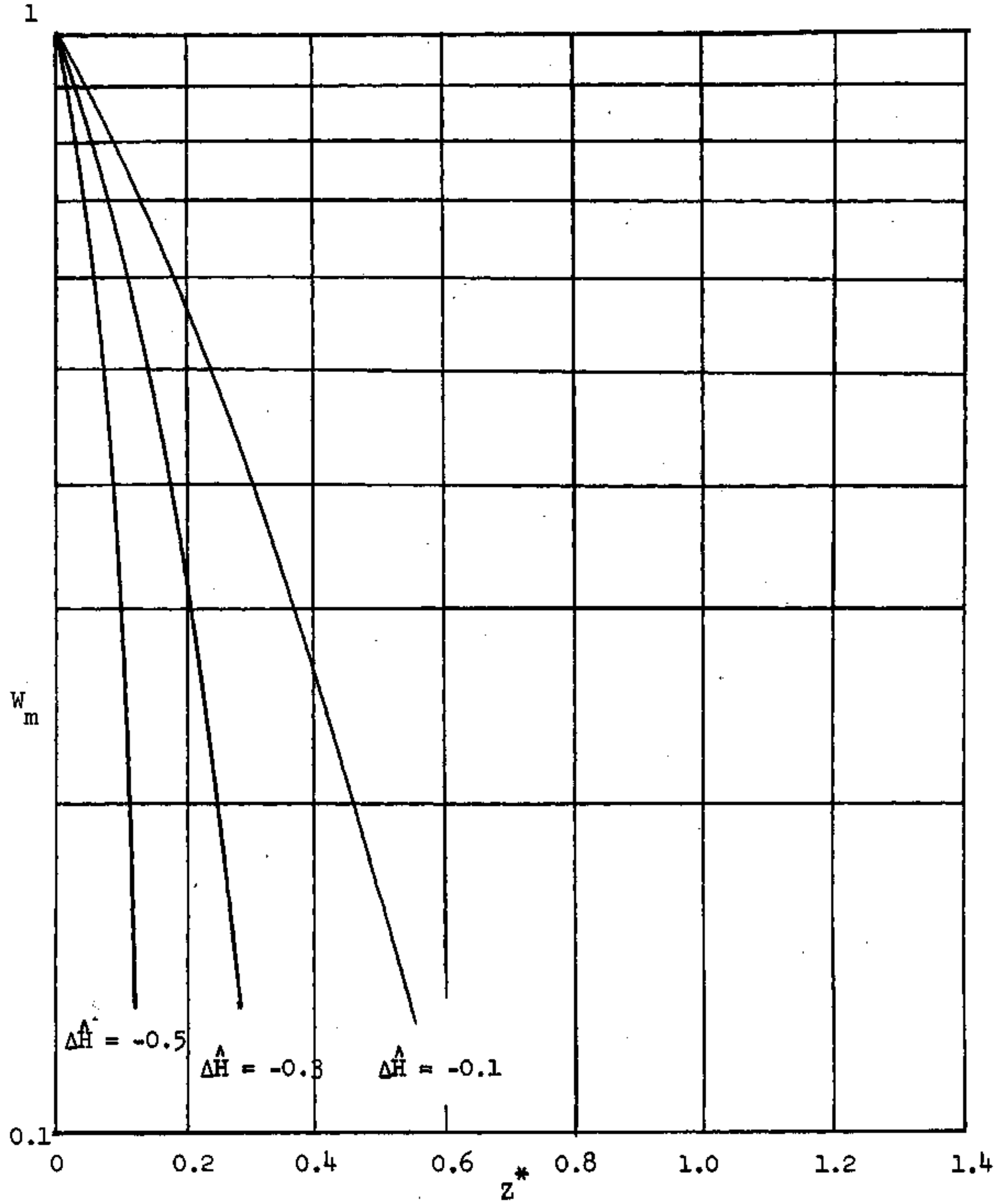


Figure 19. **Liquid Correlation**
$\alpha Pr/Sc \geq 2.5$
$E_v = 21$

temperature is found closer to the centerline. This temperature profile can, in extreme cases, cause pronounced concentration gradients across the tube. For low values $\alpha Pr/Sc$ the maximum temperature is found near the centerline, and a flattening of the concentration profile occurs. Most of the results presented here consider viscous heating to be negligible. The effect of the Eckert number is shown in Table 3. Since the Eckert numbers involve the absolute temperature and the conversion factor for mechanical to thermal units in the denominator, the values in Table 3 represent relatively high values of this number.

Heat Transfer

One of the major purposes of considering the heat-transfer problem was to establish confidence in the numerical scheme. However, a secondary purpose was to obtain results for variable property heat transfer to non-Newtonian fluids.

Craig (19) has discussed heat transfer to fluids with variable viscosity and has shown that two parameters exist for this problem, $\Delta H_v'/R'$ and T_w'/T_i'. It was also shown that an effective correlating parameter is

$$E_v = \frac{H_v'}{\mathcal{R}'} \left(\frac{T_i' - T_w'}{T_w' T_i'}\right) \qquad \text{V-3}$$

The assumptions involved in this approximation are discussed in Appendix C. Several runs were made for $E_v = 3$ and $T_w'/T_i' = 1$, 1.1 and 1.2. The results differed by less than one-half of one per cent, indicating that E_v is an effective correlating parameter. Results presented here in Appendix G are for values of E_v equal to 0, 1, 2, and 3. These values

Table 3. Effect of Viscous Heating
α = 1,000 Sc = 1,000 E_a = 21
Pr = 10

E_c = 0.0		E_c = 0.001			E_c = 0.01		
W_m	Z^*	W_m	T_m	Z^*	W_m	T_m	Z^*
0.898	0.057	0.898	1.0002	0.057	0.895	1.0017	0.057
0.799	0.123	0.799	1.0003	0.123	0.797	1.0033	0.119
0.595	0.303	0.600	1.0007	0.295	0.599	1.0067	0.274
0.400	0.565	0.401	1.0012	0.557	0.398	1.0110	0.504
0.194	1.081	0.197	1.0019	1.049	0.197	1.0173	0.905
0.088	1.671	0.087	1.0027	1.638	0.087	1.0233	1.364

represent viscosity ratios of about 2.7, 7.4 and 20 for Newtonian fluids. A limited amount of heat transfer results for developing flow are presented in Table 60 of Appendix G. The heat transfer results are illustrated in Figures 20, 21, 22 and 23.

Mass Transfer

The mass-transfer problem was not studied extensively for variable properties. Since non-Newtonian fluids normally involve very low concentrations, variable property solutions are probably not highly significant. In addition, the Lévêque approximation provides an extension of the solution to very low concentrations and the behavior of the energy equation provides a good insight into the behavior of the diffusion equation.

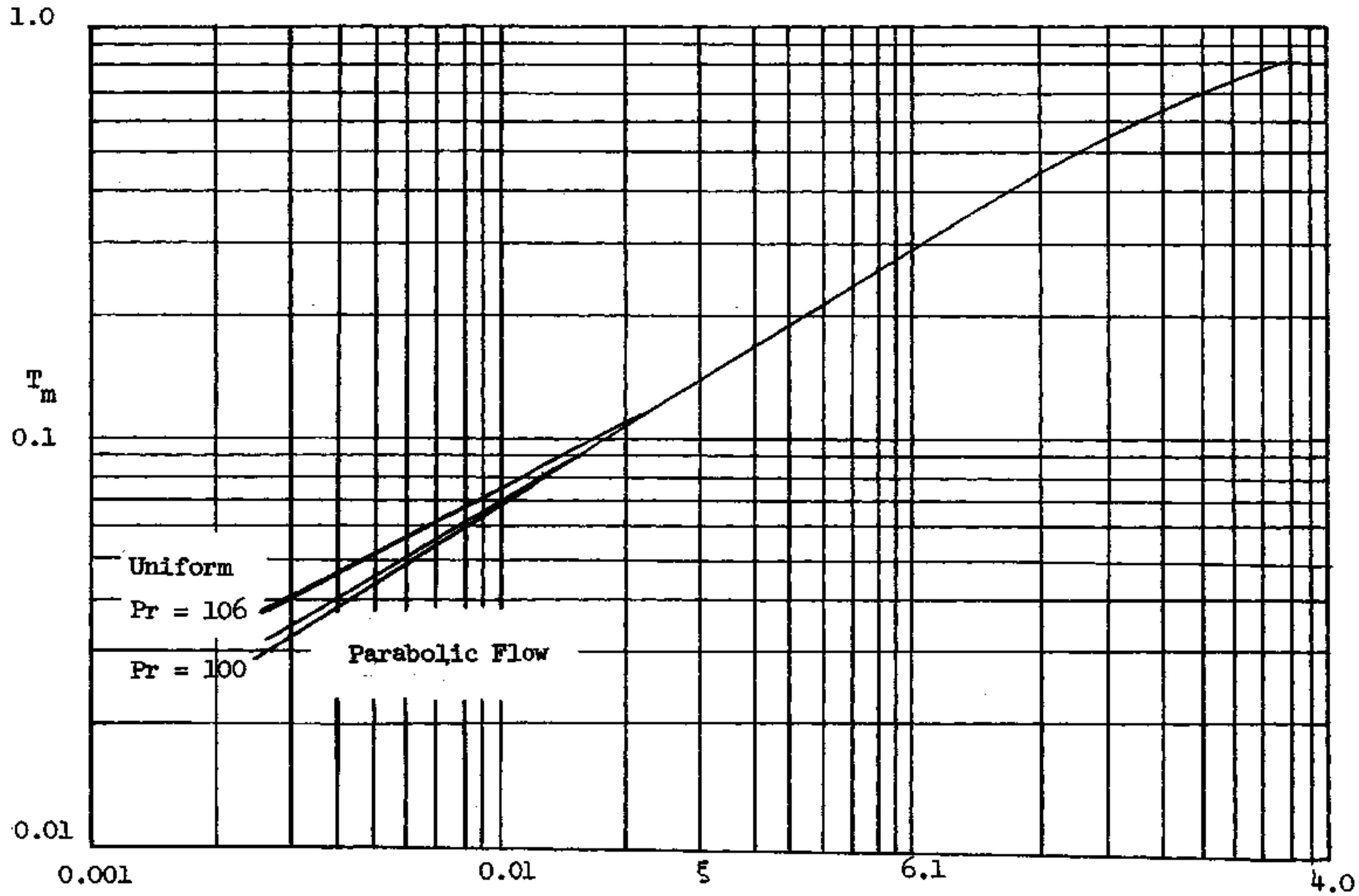


Figure 20. Effect of Developing Flow on the Heat-Transfer Problem
$n = 1.5$

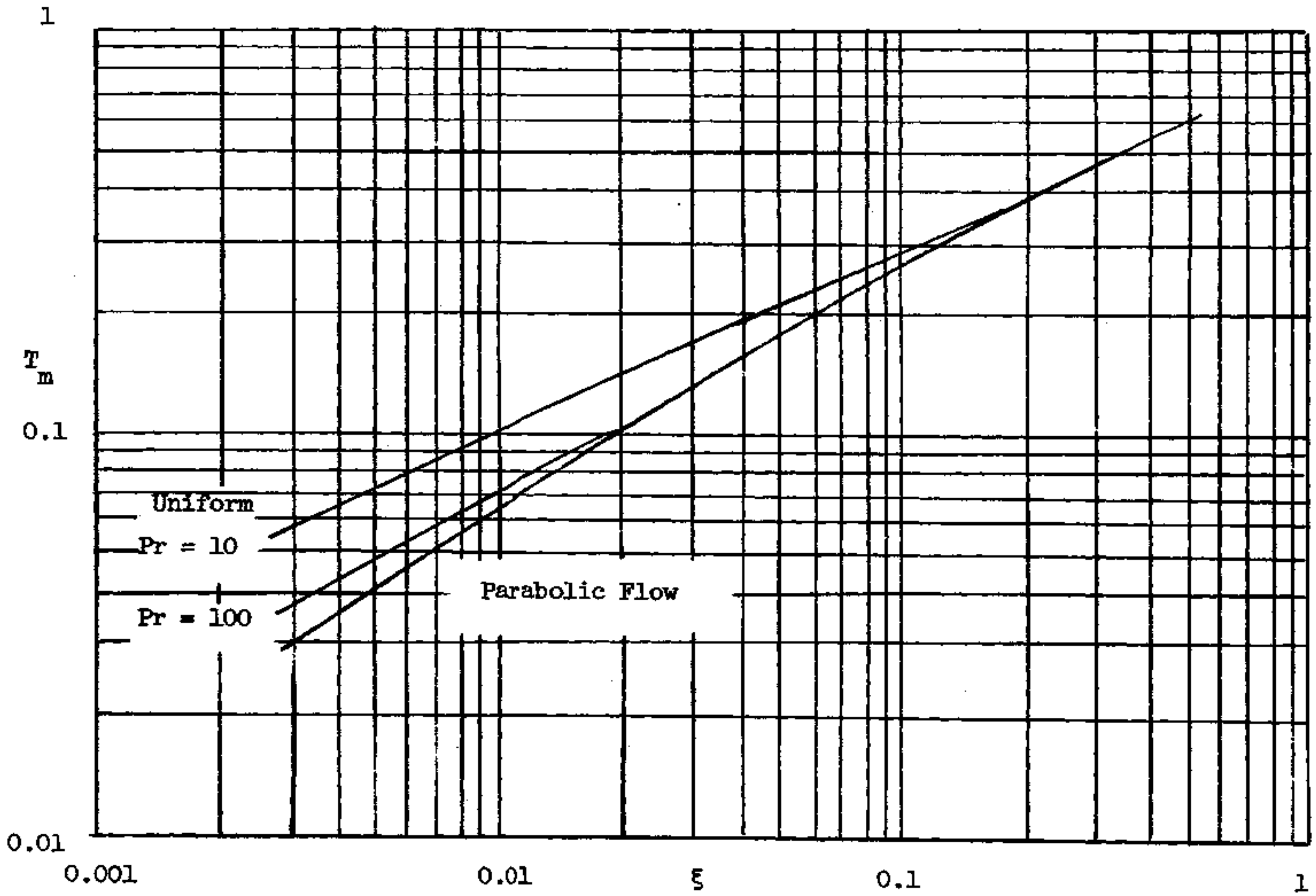


Table 21. Effect of Developing Flow on the Heat-Transfer Problem
$n = 0.2$

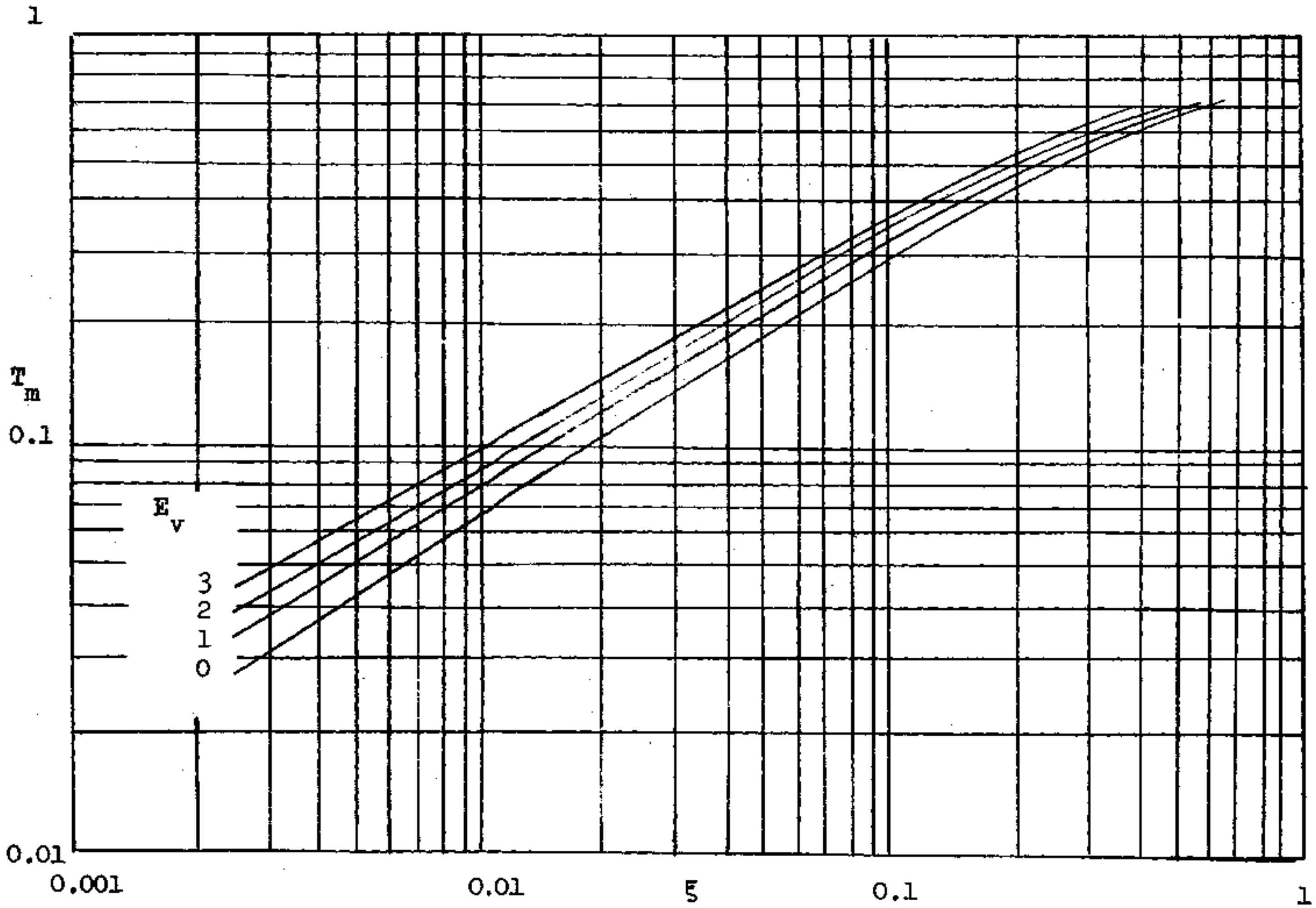


Figure 22. Effect of Variable Viscosity on the Heat-Transfer Problem
n = 1.0

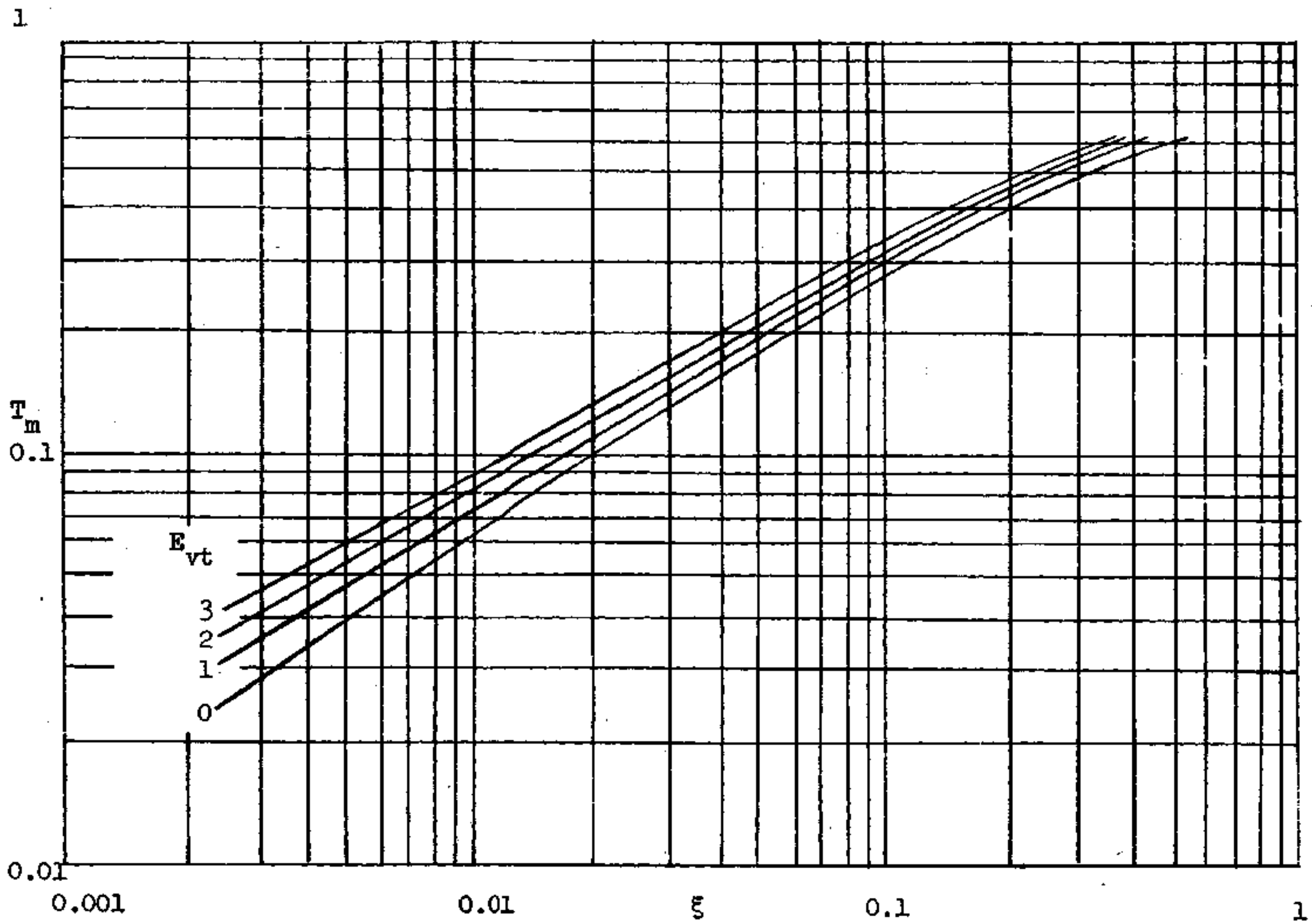


Figure 23. Effect of Variable Rheological Properties on the Heat-Transfer Problem -- n = 0.2.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Based on the results of this work, the following conclusions are reached:

1. The computer solution presented in this work is an accurate solution for the problem of a reacting non-Newtonian fluid flowing in laminar flow in a vertical tube with variable physical properties and heat effects.

2. The ratio, α/Sc, can be used to characterize the effect of a developing velocity profile. For values of this ratio less than n, the effect of the developing velocity profile is small.

3. The "plug flow" and "parabolic flow" simple models can be used as limiting cases for several important areas of interest. They provide convenient methods for correlating the results of the numerical scheme.

4. The analytical solutions presented in this work give valid solutions for situations well out of the thermal entrance region of the tube.

5. The parameters, $\alpha Pr/Sc$, $\Delta\hat{H}$, and E_a provide good correlations for the important case of exothermic reactions.

Recommendations for extensions of this work are:

1. The use of the numerical scheme presented here to cover other cases of interest.

2. The use of models other than the power-law model to describe the rheological properties of non-Newtonian fluids.

3. The use of other boundary conditions with the energy equation such as constant heat flux at the wall.

4. An experimental study of the tubular reactor problem with emphasis on heat effects.

APPENDICES

APPENDIX A

NUMERICAL METHODS

A numerical solution to a system of partial differential equations is obtained by replacing the derivatives with finite difference representations. The difference equations form a system of linear algebraic equations which can be solved by straightforward arithmetic operations.

The equations to be solved are:

The axial component of the equation of motion

$$\rho\left(v\,\frac{\partial u}{\partial r} + u\,\frac{\partial u}{\partial z}\right) = -\,\frac{dP}{dz} + \frac{\partial^2 u}{\partial r^2} + \left(\frac{\eta}{r} + \frac{\partial \eta}{\partial r}\right)\frac{\partial u}{\partial r} + \theta\rho \qquad \text{II-5}$$

The energy equation

$$\rho C_p\left(v\,\frac{\partial T}{\partial r} + u\,\frac{\partial T}{\partial z}\right) = \frac{1}{Pr}\left(k\,\frac{\partial^2 T}{\partial r^2} + \left(\frac{k}{r} + \frac{\partial k}{\partial r}\right)\frac{\partial T}{\partial r}\right.$$

$$-\,\frac{1}{Sc}\left(\Delta\hat{H}\left(\frac{\partial^2 w}{\partial r^2} + \left(\frac{D}{r} + \frac{\partial D}{\partial r}\right)\frac{\partial w}{\partial r}\right) + D\,\frac{\partial w}{\partial r}\,\frac{\partial \Delta\hat{H}}{\partial r}\right)$$

$$\left. + \,\Delta\hat{H}\rho\left(v\,\frac{\partial w}{\partial r} + u\,\frac{\partial w}{\partial z}\right) + Ec\;Kv\left[\left(\frac{\partial u}{\partial r}\right)^2\right]^{\frac{n+1}{n}} \qquad \text{II-8}$$

The diffusion equation

$$\left(v\,\frac{\partial w}{\partial r} + u\,\frac{\partial w}{\partial z}\right) = \frac{1}{Sc}\left(\frac{\partial^2 w}{\partial r^2} + \left(\frac{D}{r} + \frac{\partial D}{\partial r}\right)\frac{\partial w}{\partial r} - 16\alpha\rho K_r w\right) \qquad \text{II-11}$$

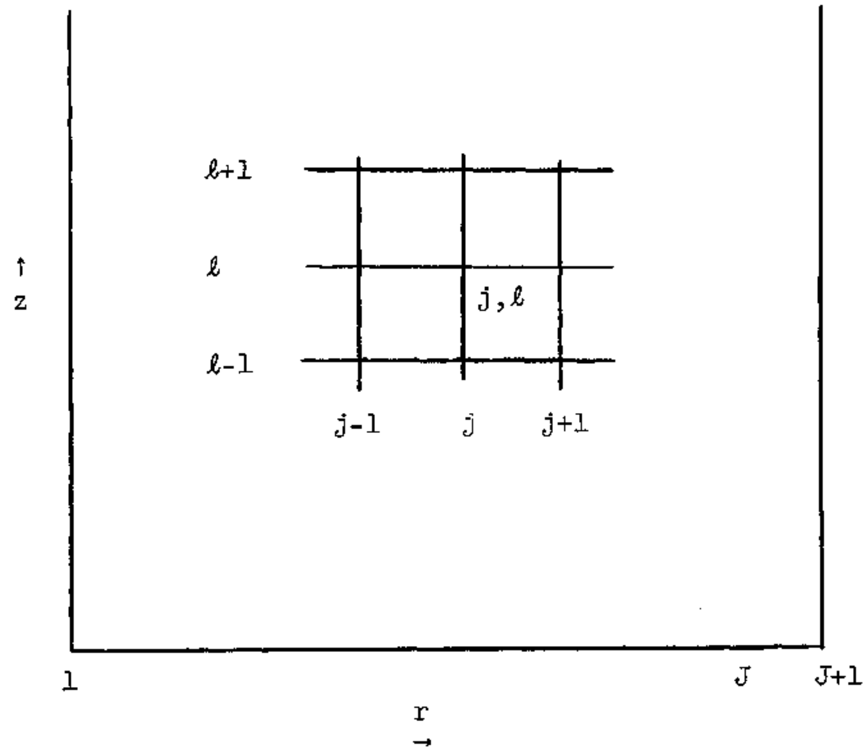
The continuity equation

$$\frac{1}{r}\,\frac{\partial}{\partial r}\,(r\rho v) + \frac{\partial}{\partial z}\,(\rho u) = 0 \qquad \text{II-2}$$

and the overall mass balance

$$\int_0^{\frac{1}{2}} \frac{\partial}{\partial z} (\rho\ ru)\ dr = \text{constant} \qquad \text{II-13}$$

A rectangular, semi-infinite grid system is superimposed on the region $0 \le r \le \frac{1}{2}$, $z \ge 0$ in the usual fashion. A general point is denoted by ${}_{j,\ell}$ where j denotes the radial index and ℓ denotes the axial index. A general unknown function, Ψ, evaluated at the point ${}_{j,\ell}$ is denoted by $\Psi_{j,\ell}$. The radial direction is divided into J divisions. The centerline is denoted by $j = 1$ while the wall is denoted by $j = J + 1$.



The finite difference representations used in this work are

$$\left(\frac{\partial\Psi}{\partial z}\right)_{j,\ell} = \frac{\Psi_{j,\ell+1} + \Psi_{j,\ell-1}}{\Delta z} \qquad \text{A-1}$$

$$\left(\frac{\partial\Psi}{\partial r}\right)_{j,\ell} = \frac{\Psi_{j+1,\ell+1} - \Psi_{j-1,\ell+1}}{2\Delta r} \qquad \text{A-2}$$

$$\left(\frac{\partial\Psi}{\partial r}\right)_{j,\ell} = \frac{\Psi_{j+1,\ell} - \Psi_{j-1,\ell}}{2\Delta r} \qquad \text{A-3}$$

(evaluation of η)

$$\left(\frac{\partial\Psi}{\partial r}\right)_{j,\ell} = \frac{3\Psi_{j,\ell+1} - 4\Psi_{j-1,\ell+1} + \Psi_{j-2,\ell+1}}{2\Delta r} \qquad \text{A-4}$$

(evaluation of boundary conditions)

$$\left(\frac{\partial^2\Psi}{\partial r^2}\right)_{j,\ell} = \frac{\Psi_{j+1,\ell+1} - 2\Psi_{j,\ell+1} + \Psi_{j-2,\ell+1}}{\Delta r^2} \qquad \text{A-5}$$

These representations are derived and discussed in standard numerical mathematics texts such as that by Lapidus (41). Bodoia (40) and Wilkins (15) have applied them to systems of equations similar to the one found here.

Concentration Profile

Substitution of these representations into the diffusion equation II-11 gives on rearrangement

$$A_j\, w_{j-1,\ell+1} + B_j\, w_{j,\ell+1} + C_j\, w_{j+1,\ell+1} = F_j \qquad \text{A-6}$$

where

$$A_j = \frac{D_{j,\ell}}{Sc\Delta r^2} - \frac{D_{j,\ell}}{2Scr_j\Delta r} - \frac{D_{j+1,\ell} - D_{j-1,\ell}}{4Sc\Delta r^2} + \frac{(\rho v)_{j,\ell}}{2\Delta r}$$

$$B_j = \frac{-2D_{j,\ell}}{Sc\ \Delta r^2} - \frac{(\rho u)_{j,\ell}}{\Delta z}$$

$$C_j = \frac{D_{j,\ell}}{Sc\ \Delta r^2} + \frac{D_{j,\ell}}{2Sc\ r_j\ \Delta r} + \frac{D_{j+1,\ell} - D_{j-1,\ell}}{4Sc\ \Delta r^2} - \frac{(\rho v)_{j,\ell}}{2\Delta r}$$

$$F_j = 16\alpha(\rho K_r w)_{j,\ell} - \frac{(\rho u w)_{j,\ell}}{\Delta z}$$

The boundary conditions at $r = 0$ and $r = \frac{1}{2}$ become

$$3w_{1,\ell+1} - 4w_{2,\ell+1} + w_{3,\ell+1} = 0 \qquad \text{A-7}$$

and

$$w_{J-1,\ell+1} - 4w_{J,\ell+1} + 3w_{J+1,\ell+1} = 0 \qquad \text{A-8}$$

The system of equations can then be written

$$\begin{aligned}
&B_1 w_{1,\ell+1} + C_1 w_{2,\ell+1} &&= F_1 \\
&A_2 w_{1,\ell+1} + B_2 w_{2,\ell+1} + C_2 w_{3,\ell+1} &&= F_2 \\
&\qquad A_3 w_{2,\ell+1} + B_3 w_{3,\ell+1} + C_3 w_{4,\ell+1} &&= F_3 \\
&\qquad \vdots && \\
&\qquad\qquad A_{J+1} w_{J,\ell+1} + B_{J+1} w_{J+1,\ell+1} &&= F_{J+1}
\end{aligned}$$

where

$$\begin{aligned}
B_1 &= A_2 - 3\,C_2 & \qquad A_{J+1} &= B_J + 4\,A_J \\
C_1 &= B_2 + 4\,C_2 & \qquad B_{J+1} &= C_J - 3\,A_J \\
F_1 &= F_2 & \qquad F_{J+1} &= F_J
\end{aligned}$$

To solve this system of equations it is necessary to eliminate only the A coefficients. The system produced by starting in the upper left hand corner is

$$
\begin{array}{llr}
BP_1\, w_{1,\ell+1} + C_1\, w_{2,\ell+1} & & = FP_1 \\
\quad BP_2 w_{2,\ell+1} + C_2 w_{3,\ell+1} & & = FP_2 \\
\quad \vdots & & \\
 & BP_{J+1} w_{J+1,\ell+1} & = FP_{J+1}
\end{array}
$$

where

$$BP_1 = B_1 \text{ and } FP_1 = F_1$$

and

$$BP_j = B_j - A_j C_{j-1}/BP_{j-1}$$

$$FP_j = F_j - A_j FP_{j-1}/BP_{j-1}$$

The concentration $w_{J+1,\ell+1}$ is found to be

$$w_{J+1,\ell+1} = FP_{J+1}/BP_{J+1}$$

The remaining concentrations are then determined by

$$w_{j,\ell+1} = (FP_j - C_j\, w_{j+1,\ell+1})/BP_j$$

Temperature Profile

The finite difference representation of the energy equation (II-8) becomes

$$A_j T_{j-1,\ell+1} + B_j T_{j,\ell+1} + C_j T_{j+1,\ell+1} = F_j \qquad \text{A-9}$$

where

$$A_j = \frac{k_{j,\ell}}{Pr\ \Delta r^2} - \frac{k_{j,\ell}}{2r_j\ Pr\Delta r} - \frac{k_{j+1,\ell} - k_{j-1,\ell}}{2Pr\ \Delta r^2} + \frac{(\rho C_p v)_{j,\ell}}{2\Delta r}$$

$$B_j = \frac{-2k_{j,\ell}}{Pr\ \Delta r^2} - \frac{(\rho C_p v)_{j,\ell}}{\Delta z}$$

$$C_j = \frac{k_{j,\ell}}{Pr\ \Delta r^2} + \frac{k_{j,\ell}}{2r_j Pr\ \Delta r} + \frac{k_{j+1,\ell} - k_{j-1,\ell}}{2Pr\ \Delta r^2} - \frac{(\rho C_p v)_{j,\ell}}{2\Delta r}$$

$$F_j = -\frac{(\rho C_p uT)_{j,\ell}}{\Delta z} + (\Delta\hat{H}\rho)_{j,\ell}\Big[v_{j,\ell}\frac{w_{j+1,\ell} - w_{j-1,\ell}}{2\Delta r}$$

$$+ u_{j,\ell}\frac{w_{j,\ell+1} - w_{j,\ell}}{\Delta z}\Big] - \Big[\Delta\hat{H}_{j,\ell}(D_{j,\ell}\frac{w_{j+1,\ell} - 2w_{j,\ell} + w_{j-1,\ell}}{\Delta r^2}$$

$$+ \frac{D_{j,\ell}}{r_j} + \frac{D_{j+1,\ell} - D_{j-1,\ell}}{2\Delta r}) + D_{j,\ell}\frac{w_{j+1,\ell} - w_{j-1,\ell}}{2\Delta r}\ x$$

$$\frac{\Delta\hat{H}_{j+1,\ell} - \Delta\hat{H}_{j-1,\ell}}{2\Delta r}\Big]/Sc - Ec\ K_{v_{j,\ell}}\Big[ABS(\frac{u_{j+1,\ell} - u_{j-1,\ell}}{2\Delta r})\Big]^{n+1}/Pr$$

The boundary conditions at $r = 0$ and $r = \frac{1}{2}$ become

$$3T_{1,\ell+1} - 4T_{2,\ell+1} + T_{3,\ell+1} = 0 \qquad \text{A-10}$$

and

$$T_{J+1,\ell+1} = T_w \qquad \text{A-11}$$

The system of equations then becomes

$$B_1 T_{1,\ell+1} + C_1 T_{2,\ell+1} = F_1$$

$$A_2 T_{1,\ell+1} + B_2 T_{2,\ell+1} + C_2 T_{3,\ell+1} = F_2$$

$$\cdot \quad \cdot \quad \cdot$$

$$A_J T_{J-1,\ell+1} + B_J T_{J,\ell+1} + C_J T_{J+1,\ell+1} = F_J$$

$$T_{J+1,\ell+1} = T_w$$

where

$$B_1 = A_2 - 3C_2$$

$$C_1 = B_2 + 4C_2$$

$$F_1 = F_2$$

If this system of equations is solved by eliminating the A coefficients, the computational algorithm is

$$BP_1 = B_1$$

$$FP_1 = F_1$$

$$BP_j = B_j - A_j\, C_j / BP_{j-1}$$

$$FP_j = F_j - A_j FP_{j-1} / BP_{j-1}$$

for

$$2 \le j \le J$$

Finally,

$$T_{J+1,\ell+1} = T_w$$

and

$$T_{j,\ell+1} = (FP_j - C_j T_{j+1,\ell+1})/BP_j$$

for

$$J \geq j \geq 1$$

Axial Velocity Profile

Before putting the equation of motion into finite difference form, it is advantageous to consider the functional form of η. The power-law assumption states that

$$\eta = K_v \left|\frac{\partial u}{\partial r}\right|^{n-1} \qquad \text{A-12}$$

if $\frac{\partial u}{\partial r} > 0$, then

$$\eta = K_v \left(\frac{\partial u}{\partial r}\right)^{n-1}$$

and

$$\frac{\partial \eta}{\partial r} = K_v (n-1) \left(\frac{\partial u}{\partial r}\right)^{n-2} \frac{\partial^2 u}{\partial r^2} + \left(\frac{\partial u}{\partial r}\right)^{n-1} \frac{\partial K_v}{\partial r}$$

Finally

$$\eta \frac{\partial^2 u}{\partial r^2} + \left(\frac{\eta}{r} + \frac{\partial \eta}{\partial r}\right) \frac{\partial u}{\partial r} = \left(\frac{\partial u}{\partial r}\right)^{n-1} \left[K_v \frac{\partial^2 u}{\partial r^2} + \frac{K_v}{r} \frac{\partial u}{\partial r} + K_v (n-1) \frac{\partial^2 u}{\partial r^2} + \frac{\partial K_v \partial u}{\partial r \partial r}\right]$$

$$= \left(\frac{\partial u}{\partial r}\right)^{n-1} \left[\eta\, K_v \frac{\partial^2 u}{\partial r^2} + \left(\frac{K_v}{r} + \frac{\partial K_v}{\partial r}\right) \frac{\partial u}{\partial r}\right]$$

If $\frac{\partial u}{\partial r} < 0$, then

$$\eta = K_v \left(- \frac{\partial u}{\partial r}\right)^{n-1}$$

and

$$\frac{\partial \eta}{\partial r} = K_v(n-1)(-\frac{\partial u}{\partial r})^{n-2}(-\frac{\partial^2 u}{\partial r^2}) + (-\frac{\partial u}{\partial r})^{n-1}\frac{\partial K_v}{\partial r}$$

Finally,

$$\frac{\partial^2 u}{\partial r^2} + (\frac{\eta}{r} + \frac{\partial \eta}{\partial r})\frac{\partial u}{\partial r} = (-\frac{\partial u}{\partial r})^{n-1}\left[K_v \frac{\partial^2 u}{\partial r^2} + \frac{K_v}{r}\frac{\partial u}{\partial r} + K_v(n-1)\frac{\partial^2 u}{\partial r^2} + \frac{\partial K_v}{\partial r}\frac{\partial u}{\partial r}\right]$$

$$= (-\frac{\partial u}{\partial r})^{n-1}\left[K_v \frac{\partial^2 u}{\partial r^2} + (\frac{K_v}{r} + \frac{\partial K_v}{\partial r})\frac{\partial u}{\partial r}\right]$$

Therefore the equation takes the following form

$$\rho(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}) = -\frac{dP}{dz} + \left|\frac{\partial u}{\partial r}\right|^{n-1}\left[\eta K_v \frac{\partial^2 u}{\partial r^2} + (\frac{K_v}{r} + \frac{\partial K_v}{\partial r})\frac{\partial u}{\partial r}\right] + \theta\rho \qquad \text{A-13}$$

The finite difference representation of this equation is then

$$A_j u_{j-1,\ell+1} + B_j u_{j,\ell+1} + C_j u_{j+1,\ell+1} + D_j P_{\ell+1} = F_j \qquad \text{A-14}$$

where

$$\eta^* = \left[\frac{u_{j+1,\ell} - u_{j-1,\ell}}{2\Delta r}\right]^{n-1}$$

$$A_j = \left[\frac{nK_{v\,j,\ell}}{\Delta r^2} + \frac{K_{v\,j,\ell}}{2r_j\Delta r} - \frac{K_{v\,j+1,\ell} - K_{v\,j-1,\ell}}{4\Delta r^2}\right]\eta^* - \frac{(\rho v)_{j,\ell}}{2\Delta r}$$

$$B_j = -\frac{2nK_{v\,j,\ell}}{\Delta r^2}\eta^* - \frac{(\rho u)_{j,\ell}}{\Delta z}$$

$$C_j = \left[\frac{nK_{v\,j,\ell}}{\Delta r^2} + \frac{K_{v\,j,\ell}}{2r_j\Delta r} + \frac{K_{v\,j+1,\ell} - K_{v\,j-1,\ell}}{4\Delta r^2}\right]\eta^* + \frac{(\rho v)_{j,\ell}}{2\Delta r}$$

$$D_j = -\frac{1}{\Delta z}$$

$$F_j = -\frac{P_\ell}{\Delta z} - \frac{(\rho u^2)_{j,\ell}}{\Delta z} - \theta\rho_{j,\ell}$$

The boundary conditions at $r = 0$ and $r = \frac{1}{2}$ are

$$3u_{1,\ell+1} + 4u_{2,\ell+1} + u_{3,\ell+1} = 0 \qquad \text{A-15}$$

$$u_{J+1,\ell+1} = 0 \qquad \text{A-16}$$

There are J unknown velocities and the unknown pressure of J + 1 unknowns. There are j - 1 difference equations from the equation of motion and one boundary condition or J equations. The remaining equation, the overall mass balance, will be shown later to be

$$\sum_{j=1}^{J+1} G_j u_{j,\ell+1} = \sum_{j=1}^{J+1} G_j u_{j,\ell} \qquad \text{A-17}$$

where

$$G_1 = R_2 \rho_2 / 4$$

$$G_2 = 3R_2 \rho_2 / 4$$

and

$$G_j = R_j \rho_j$$

for

$$3 \leq j \leq J$$

The system of equations then becomes

$$G_1 u_{1,\ell+1} + G_2 u_{2,\ell+1} + G_3 u_{3,\ell+1} \cdot \cdot \cdot \; G_J u_{J,\ell+1} \qquad = F_0$$

$$B_1 u_{1,\ell+1} + C_1 u_{2,\ell+1} \qquad + D_1 P_{\ell+1} = F_1$$

$$A_2 u_{1,\ell+1} + B_2 u_{2,\ell+1} + C_2 u_{2,\ell+1} \qquad + D_2 P_{\ell+1} = F_2$$

$$A_3 u_{3,\ell+1} + B_3 u_{3,\ell+1} + C_3 u_{3,\ell+1}$$

$$\cdot \quad \cdot \quad \cdot$$

$$A_J u_{J-1,\ell+1} + B_J u_{J,\ell+1} + D_J P_{\ell+1} = F_J$$

where

$$B_1 = A_2 - 3C_2$$

$$C_1 = B_2 + 4C_2$$

$$F_o = \sum_{j=1}^{J} G_j u_{j,\ell}$$

Considering only Equations 1 through J, the A coefficients can be eliminated in the usual fashion to form

$$G_1 u_{1,\ell+1} + G_{2,\ell+1} + G_{3,\ell+1} \cdot \cdot \cdot \qquad = F_o$$

$$BP_1 u_{1,\ell+1} + C_2 u_{2,\ell+1} \qquad + DP_1 P_{\ell+1} = F_1$$

$$BP_2 u_{2,\ell+1} + C_2 u_{3,\ell+1} \qquad + DP_2 P_{\ell+1} = F_2$$

$$\cdot \quad \cdot \quad \cdot$$

$$BP_J u_{J,\ell+1} \qquad + DP_J P_{\ell+1} = F_J$$

where

$$BP_1 = B_1 \qquad DP_1 = D_1 \qquad FP_1 = F_1$$

and

$$BP_j = B_j - A_j C_j / BP_{j-1} \qquad DP_j = D_j - A_j DP_{j-1}/BP_{j-1}$$

$$FP_j = F_j - A_j FP_{j-1}/BP_{j-1}$$
$$2 \quad j \quad J$$

The B coefficients are then eliminated to form

$$G_1 u_{1,\ell+1} + G_2 u_{2,\ell+1} + \cdot \cdot \cdot$$

$$CPP_1 u_{2,\ell+1} + G_3 u_{3,\ell+1} + \cdot \cdot \cdot \qquad + DPP_1 P_{\ell+1} = F_1$$

$$CPP_2 u_{3,\ell+1} + G_4 u_{4,\ell+1} \cdot \cdot \cdot \qquad + DPP_2 P_{\ell+1} = F_2$$

$$\cdot \cdot \cdot$$

$$DPP_J P_{\ell+1} = F_J$$

where

$$CPP_1 = G_2 - G_1 C_1/BP_1$$

$$DPP_1 = - G_1 DP_1/BP_1$$

$$FPP_1 = F_o - G_1 FP_1/BP_1$$

and

$$CPP_j = G_{j+1} - CPP_{j-1} C_j/BP_j$$

$$DPP_j = DPP_{j-1} - CPP_{j-1} DP_j/BP_j$$

$$FPP_j = FPP_{j-1} - CPP_{j-1} FP_j/BP_j$$

for $\qquad 1 \le j \le J$

Then the solution is found to be

$$P_{\ell+1} = FPP_J / DPP_J$$

$$u_{J,\ell+1} = (FPP_{J-1} - DP_{J-1} P_{\ell+1}) / CPP_{J-1}$$

and $\qquad u_{j,\ell+1} + (FP_{j-1} - DP_j P_{\ell+1} - C_j u_{j+1,\ell+1}) / BP_j$

for $\qquad 1 \le j \le J\text{-}1$

Radial Velocity Profile

The radial velocity components are determined from the microscopic form of the continuity equation. The following difference equations are used

$$\left(r \frac{\partial \rho u}{\partial z}\right)_{j,\ell} = \frac{r_j(\rho u_{j,\ell+1} - \rho u_{j,\ell}) + r_{j+1}(\rho u_{j+1,\ell+1} - \rho u_{j+1,\ell})}{2\Delta z} \qquad \text{A-18}$$

and

$$\left(\frac{\partial \rho r v}{\partial r}\right)_{j,\ell} = \frac{(\rho r v)_{j+1,\ell+1} - (\rho r v)_{j,\ell+1}}{\Delta r} \qquad \text{A-19}$$

The resulting equation is

$$v_{j+1,\ell+1} = \frac{1}{r_{j+1,\ell+1}} \Big[(\rho r v)_{j,\ell+1} + \frac{\Delta r}{2\Delta z} \Big[r_j(\rho u_{j,\ell+1} - \rho u_{j,\ell})$$

$$+ r_{j+1}(\rho u_{j+1,\ell+1} - \rho u_{j+1,\ell}) \Big] \Big] \qquad \text{A-20}$$

for $\qquad 1 \le j \le J$

where $\qquad v_{1,\ell+1} = 0$

The overall mass balance, used previously as Equation (A-17), can be derived by integrating the microscopic continuity equation from $r = 0$ to $\frac{1}{2}$. Equation (A-20) is valid for all points in the tube except the tube axis. Applying L'Hospital's rule at the tube axis gives

$$\left.\frac{\partial \rho u}{\partial z}\right|_{r=0} + 2\,\frac{\partial \rho v}{\partial r} = 0 \qquad \text{A-21}$$

or

$$v_{2,\ell+1} = \frac{\Delta r}{4\Delta z}\,(\rho u)_{2,\ell+1} + (\rho u)_{1,\ell+1} - (\rho u)_{2,\ell} - (\rho u)_{1,\ell} \qquad \text{A-22}$$

Adding this result to Equation (A-20) for each point across the tube gives Equation (A-17) as the result.

Average Concentration and Temperature

The average concentration in terms of dimensionless variables is given by

$$W_m = 8\int_0^{\frac{1}{2}} w\rho u r\;dr \qquad \text{A-23}$$

Although the heat capacity was assumed to be variable in the derivation of the equations, it was not varied in any of the actual runs. For constant heat capacity the average temperature is given by

$$T_m = 8\int_0^{\frac{1}{2}} T\rho u r\;dr \qquad \text{A-24}$$

A generalized mean temperature, valid for variable heat capacity, is discussed by Lee (14) and Wilkins (15).

Nusselt Number Calculations

The local Nusselt number is defined by

$$Nu_L = \frac{h'2R'}{k_o'} \qquad \text{A-25}$$

The local heat-transfer coefficient can be calculated by two independent methods for the heat-transfer problem. It can be determined either by the radial temperature derivative evaluated at the wall or by the axial derivative of the mean temperature. These two methods yield respectively for constant thermal conductivity

$$Nu_L = \frac{\left.\frac{\partial T}{\partial r}\right|_{r = \frac{1}{2}}}{(1 - T_m)} \qquad \text{A-26}$$

$$= \frac{(3T_{J+1} - 4T_J + T_{J-1})}{2\Delta r(1 - T_m)} \qquad \text{A-27}$$

and

$$Nu_L = \frac{Pr\partial T_m}{4\partial z} \qquad \text{A-28}$$

$$\approx \frac{Pr(T_{\ell+1} - T_{\ell-1})}{4\ \Delta z} \qquad \text{A-29}$$

A measure of the accuracy of the solution is then given by

$$\text{Error} = \frac{Nu_{L\ axial} - Nu_{L\ radial}}{Nu_{L\ axial} + Nu_{L\ radial}} \qquad \text{A-30}$$

For the tubular problem when there is no viscous dissipation and the heat of reaction is constant, the ratio of the heat exchanged with the environment to that generated by the reaction can be determined by two similar methods. This ratio becomes

$$Q_{radial} = \frac{4\left.\frac{\partial T}{\partial r}\right|_{r = \frac{1}{2}}}{Pr\frac{\partial W}{\partial z}} \qquad \text{A-31}$$

$$Q_{axial} = 1 - \frac{\frac{\partial T}{\partial z}}{-\Delta\hat{H}\frac{\partial W}{\partial z}} = 1 - \frac{\partial T}{\Delta\hat{H}\partial W} \qquad \text{A-32}$$

Definitions of other Nusselt numbers are often useful. That based on the initial temperature difference is defined by

$$Nu_o = \frac{h_o' 2R'}{k_o'} \qquad \text{A-33}$$

$$= \frac{Pr\ T_m}{4z} \qquad \text{A-34}$$

The arithmetic mean Nusselt number is defined by

$$Nu_{AM} = \frac{h_{AM}' 2R'}{k'} \qquad \text{A-35}$$

$$= \frac{4\ Pr\ T_m}{z(2 - T_m} \qquad \text{A-36}$$

and finally that based on the log mean temperature difference is

$$Nu_{LN} = \frac{h_{LN}' 2R'}{k'} \qquad \text{A-37}$$

$$= -\frac{Pr\ \ln(1 - T_m)}{4z} \qquad \text{A-38}$$

APPENDIX B

COMPUTER PROGRAM

The calculations were made on a Burroughs B-5500 Information Processing System operated by the Rich Electronic Computer Center at the Georgia Institute of Technology. The programming language was Extended Algol 60.

Outline

The computer program has been divided into a number of sections. These sections are separated by several spaces and each section is preceded by a COMMENT statement which describes the function of that section.

The first section contains the declarations and the input and output lists and formats. In the second section data is read into the program and initial values are assigned to the variables.

The physical properties are evaluated in the third section. The fourth through the seventh sections contain the calculations of the concentration, temperature, axial and radial velocity profiles. These profiles are averaged in the eighth section and the Nusselt numbers and error terms calculated in the ninth section.

The last four sections contain the output procedure, the scheme for changing DR and DZ and the procedure for repeating the cycle of calculations.

Computer Program Nomenclature

In order to facilitate reading the computer program the more important variables are listed below. Many of the dummy variables are not listed.

Independent Variables

DR	....	Δr
DZ	....	Δz
R	....	r
ZP	....	z

Dependent Variables

PR1	....	P
PR2	....	P
V	....	v
U	....	u
W	....	w

Physical Properties

CP	....	C_p
HR	....	H
KR	....	K_r
KT	....	K
KV	....	K_v
N	....	n
RDF	....	D
RHO	....	ρ

```
BEGIN COMMENT  JACK WHATLEY - A REACTING NON-NEWTONIAN FLUID FLOWING
IN A VERTICAL TUBE                                                   ;

REAL DR, DT, DZ, ERR, ERRA, ERRAD, FLOW, GRAV, GZ, I, K, K1, LAMDA,
LNUSR, LNUSZ, M, N1, NUSAV, NUSLN, NUSO, TAVG, TAVG1, PR1, PR2, PRN,
RAD, RAD1, REDFR, SC, TQ, TW, UAVG, VIS1, VIS2, VIS3, WAVG, WAVG1,
Y1, Y2, Y3, ZP, ZPT                                                  ;

REAL ARRAY A, B, BP, BPP, C, CP, CPP, D, DP, DPP, F, FP, FPP, G, HR,
KR, KT, KV, N, R, RDF, V[0:250], RHO, T, U, W[0:2,0:250]             ;

LIST DATA(ALP, SC, PRN, RADFR, M, DZ, N1, FLOW)                      ;
LIST DATA1(FOR K + 1 STEP 1 UNTIL 5 DO CN[I,K])                      ;
FORMAT OUT HOW1(" ALP = ",R15.5,X10," SC = ",R15.5,X10," PRN - ",
     /," RE/FR = ",R15.5,X10," M = ",I5,X10,' INITIAL DZ = ",R15.5,
     /,"N1 = ",R15.5,X10," FLOW = ",I5,//)                           ;
LIST OUT2(FOR K ← 1 STEP 1 UNTIL 8 DO CN[K,I])                       ;
FORMAT OUT HOW2(8R15.5)                                              ;
LIST OUT3(AXZ, GZ, LAMDA, UAVG, WAVG, TAVG, PR2, GRAV, ERR, K, DT)    ;
FORMAT OUT HOW3(" AXZ = ",R15.5," GRAETZ = ",R15.5,"LAMDA = ",R15.5
     ,/," AVG VEL = ",R15.5," AVG CONC = ",R15.5," AVG TEMP = ",
     R15.5," TOTAL PRES = ",R15.5," GRAV = ",R15.5,/," THERMAL ERR ="
     ,R15.5," TIME AFTER ", I5," STEPS IS ",R15.5)                   ;
LIST OUT4(RAD, U[2,I], W[2,I], T[2,I])                               ;
FORMAT OUT HOW4(4R25.5)                                              ;
FORMAT OUT FMT1(X30,"PHYSICAL PROPERTY CONSTANTS")                   ;
FORMAT OUT FMT2("CONC BELOW 0 ")                                     ;
```

```
FORMAT OUT FMT3(" VEL BELOW 0 ")                                    ;
FORMAT OUT FMT4(X10,"RAD",X25,"U",X25,"W",X25,"T")                  ;
FORMAT OUT FMT5(" DZ INCREASED")                                    ;

LABEL TOM, LEND                                                     ;

PROCEDURE AVGVAL                                                    ;
BEGIN
FOR I + 3 STEP 1 UNTIL M+1 DO BEGIN
     Y3 ← Y1 + (A[I] + 4XA[I-1] + A[I-2])XDR/3                      ;
     Y1 ← Y2     ;     Y2 + Y3                                      ;
END       ;     END                                                 ;

COMMENT START PROGRAM - SET INITIAL VALUES                          ;
READ(FIN,/,DATA)                                                    ;
FOR I ← 1 STEP 1 UNTIL 8 DO READ(FIN,/,DATA1)                       ;
CLOSE(FIN,RELEASE)                                                  ;
TQ ← TIME(2)       ;       ZP ← DZ                                  ;
PR1 ← 0            ;       DR ← 0.50/M                              ;
WAVG1 ← 1          ;       TAVG1 ← 1                                ;
FOR I ← 1 STEP 1 UNTIL M + 1 DO BEGIN
     R[I] ← (I-1)xDR                                                ;
     V[I] ← 0                                                       ;
     T[1,I] ← 1    ;       [1,I] ← 1                                ;
     IF FLOW = 0 THEN U[1,I] ← (3xN1 + 1)x(1 - (2xR[I])*((M1 + 1)/N1))
        /(N1 + 1)     ELSE       U[1,I] ← 1                         ;
END
```

```
WRITE(FOUT,HOW1,DATA)                                                   ;
WRITE(FOUT,FMT1)                                                        ;
FOR I ← 1 STEP 1 UNTIL 5 DO WRITE(FOUT,HOW2,OUT2)                       ;

COMMENT  EVALUATION OF PHYSICAL PROPERTIES                              ;
COMMENT  IN THIS SECTION THE EQUATIONS VARY FROM PROGRAM TO PROGRAM
     -   HERE THE PROPERTIES ARE MERELY LISTED                          ;
TOM:
FOR I ← 1 STEP 1 UNTIL M + 1 DO BEGIN
       RHO[1,I]        RDF[I]        KR[I]
       KT[I]           CP[I]         HR[I]
       KV[I]           N[I]
END                                                                     ;

COMMENT DIFFUSION EQN
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
     VIS1 ← RDF[I]/(SCxDR*2)                                            ;
     VIS2 ← RHO[1,I]xV[I]/(2xDR)                                        ;
     VIS3 ← (RDF[I]/R[I]+(RDF[I+1]- RDF[I-1])/(2xDR))/SC                ;
     A[I] ← VIS1 + VIS2 - VIS3                                          ;
     B[I] ← -2xVIS1 - RHO[1,I]x U[1,I]/DZ                               ;
     C[I] ← VIS1 - VIS2 + VIS3                                          ;
     F[I] ← -RHO[1,I]xU[1,I]xW[1,I]/DZ + 16xALPxRHO[1,I]xKR[I]xW[1,I]   ;
END                                                                     ;
BP[1]← A[2] - 3xC[2] ; C[1] ← B[2] + 4xC[2]  ;  FP[k] ← F[2]            ;
A[M+1] ← B[M] + 4xA[M] ; B[M+1] ← C[M] - 3xA[M];F[M+1] ← F[M]           ;
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
```

```
     BP[I] ← B[I] - A[I]xC[I-1]/BP[I-1]                                 ;
     FP[I] ← F[I] - A[I]xFP[I-1]/BP[I-1]                                ;
END                                                                     ;
     W[2,M+;] ← FP[M+1]/BP[M+1]                                         ;
FOR I ← M STEP -1 UNTIL 1 DO BEGIN
     W[2,I] ← (FP[I] - C[I]xW[2,I+1])/BP[I]                             ;
     IF W[2,I] ≤ 0 THEN BEGIN
          WRITE(FOUT,FMT2)                                              ;
          GO TO LEND                                                    ;
END    ;      END                                                       ;

COMMENT ENERGY EQN                                                      ;
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
     VIS1 ← KT[I]/(PRNxDR*2)                                            ;
     VIS2 ← RHO[1,I]xV[I]CP[I]/(2xDR) - KT[I]/(PRNxR[I]xDR*2)           ;
     A[I] ← VIS1 + VIS2                                                 ;
     B[I] ← -2xVIS1 - RHO[1,I]xCP[I]xU[1,I]/DZ                          ;
     C[I] ← VIS1 - VIS2                                                 ;
     F[I] ← -RHO[1,I]XU[1,I]xT[1,I]/DZ +(HR[I]x(RDF[I]w(W[1,I+1]
            -2xW[1,I] + W[1,I-1])/DR*2 + RDF[I]/R[I] +
            (RDF[I+1] - RDF[I-1])/(2xDR)) + RDF[I]x(W[1,I+1]-W[1,I-1])x
            (HR[I+1] - HR[I-1])/(4xDR*2))/SC + HR[I]xRHO[1,I]x
            (V[I]x(W[1,I+1]-W[1,I-1])/(2xDRO) + U[1,I]x(W[2,I]-W[1,I])/
            DZ) + BRxKV[I]xABS((U[1,I+1]-U[1,I-1])/(2xDR))*(N[I]+1)/PRN;
END                                                                     ;
BP[1] ← A[2]-3xC[2]  ;  C[1] ← B[2] + 4xC[2]  ;  FP[1] ← F[2]           ;
```

```
A[M+1] ← B[M] + 4xA[M] ; B[M+1] ← C[M]-3xA[M]; F[M+1] ← F[M]           ;
FOR I ← 2 STEP 1 UNTIL M + 1 DO BEGIN
     BP[I] ← B[I] - A[I]xC[I-1]/BP[I-1]                                ;
     FP[I] ← F[I] - A[I]xFP[I-1]/BP[I-1]                               ;
END                                                                    ;
     T[2,M+1] ← TW                                                     ;
FOR I ← M STEP -1 UNTIL 1 DO
     T[2,I] ← (FP[I]-C[I]xT[2,I+1])/BP[I]                              ;

COMMENT  MOMENTUM EQN                                                  ;
COMMENT  HERE THE DENSITY IS EVALUATED AT LEVEL 2                      ;
FOR I ← 1 STEP 1 UNTIL M+1 DO    RHO[2,I]
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
     VIS1 ← ABS((U[1,I+1]-U[1,I-1])/(2xDR))*(N[I]-1)                   ;
     VIS2 ← (KV[I]/R[I] + (KV[I+1]-KV[I-1])/(2xDR))/(2xDR)             ;
     VIS3 ← RHO[1,I]xV[I]/(2xDR)                                       ;
     A[I] ← VIS1x(N[I]/DR*2-VIS2) + VIS3                               ;
     B[I] ← -2xN[I]xVIS1/DR*2 - RHO[1,I]xU[1,I]/DZ                     ;
     C[I] ← VIS1x(N[I]/DR*2 + VIS2) - VIS3                             ;
     D[I] ← -1/DZ                                                      ;
     F[I] ← -RHO[1,I]x(REDFR+U[1,I]*2/DZ) - PR1/DZ                     ;
END                                                                    ;
G[1] ← 0.25xR[2]xRHO[2,2]  ;  G[2] ← 0.75xR[2]xRHO[2,2]                ;
FOR I ← 3 STEP 1 UNTIL M DO G[I] ← R[I]xRHO[2,I]                       ;
BP[1] ← A[2] -3xC[2]  ;    C[1] ← B[2] + 4xC[2]                        ;
DP[1] ← D[2]          ;    FP[1] ← F[2]                                ;
```

```
F[0] ← DRx(RHO[1,2]xU[1,1]/4 + RHO[1,2]xU[1,2]x0.75)                    ;
FOR I ← 3 STEP 1 UNTIL M DO F[0] ← F[0]+R[I]xRHO[1,I]xU[1,I]            ;
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
     BP[I] ← B[I] -A[I]xC[I-1]/BP[I-1]                                  ;
     DP[I] ← D[I] -A[I]xDP[I-1]/BP[I-1]                                 ;
     FP[I] ← F[I] -A[I]xFP[I-1]/BP[I-1]                                 ;
END                                                                     ;
CPP[1] ← G[2] - G[1]xC[1]/BP[1]                                         ;
DPP[1] ← -G[1]xDP[1]/BP[1]                                              ;
FPP[1] ← F[0] - G[1]xFP[1]/BP[1]                                        ;
FOR I ← 2 STEP 1 UNTIL M DO BEGIN
     CPP[I] ← G[I+1] - C[I]xCPP[I-1]/BP[I]                              ;
     DPP[I] ← DPP[I-1] - DP[I]xCPP[I-1]/BP[I]                           ;
     FPP[I] ← FPP[I-1] - FP[I]xCPP[I-1]/BP[I]                           ;
END                                                                     ;
     PR2 ← FPP[M]/DPP[M]  ;  U[2,M+1] ← 0                               ;
     U[2,M] ← (FPP[M-1] - DPP[M-1]xPR2)/CPP[M-1]                        ;
FOR I ← M-1 STEP -1 UNTIL 1 DO BEGIN
     U[2,I] ← (FP[I] - DP[I]xPR2 - C[I]xU[2,I+1])/BP[I]                 ;
     IF U[2,I] < 0 THEN BEGIN
            WRITE(FOUT,FMT3)                                            ;
            GO TO LEND                                                  ;
END            ;            END                                         ;

COMMENT RADIAL VELOCITIES                                               ;
V[1] ← 0                                                                ;
```

```
V[2] ← -(RHO[2,1]xU[2,1] + RHO[2,2]xU[2,2] - RHO[1,1]xU[1,1] -
     RHO[1,2]xU[1,2]xDR/(4xDZxRHO[2,2])                              ;
FOR I ← 3 STEP 1 UNTIL M + 1 DO BEGIN
V[I] ← R[I-1]xV[I-1]xRHO[2,I-1]/(R[I]xRHO[2,I]) - (U[2,I]xRHO[2,I]-
     U[1,I]xRHO[1,I])xDR/(2xDZxRHO[2,I]) - (U[2,I-1]xRHO[2,I-1]-
     U[1,I-1]xRHO[1,I-1])xDRxR[I-1]/(2xDZxRHO[2,I]xR[I])            ;
IF V[I] > V[I-1] THEN VMAX ← V[I]                                    ;
END                                                                  ;

COMMENT CAL AVGS                                                     ;
FOR I ← 1 STEP 1 UNTIL M + 1 DO A[I] ← R[I]xU[2,I]                   ;
     AVGVAL                                                          ;
     UAVG ← 8xY3                                                     ;
FOR I ← 1 STEP 1 UNTIL M + 1 DO A[I] ← R[I]xRHO[2,I]xU[2,I]xW[2,I]   ;
     AVGVAL                                                          ;
     WAVG ← 8xY3                                                     ;
FOR I ← 1 STEP 1 UNTIL M + 1 DO A[I] ← R[I]xRHO[2,I]xU[2,I]xT[2,I]   ;
     AVGVAL                                                          ;
     TAVG ← 8xY3                                                     ;
FOR I ← 1 STEP 1 UNTIL M + 1 DO A[I] ← R[I]xRHO[2,I]                 ;
     AVGVAL                                                          ;
     GRAV ← 8xY3                                                     ;
DT ← (TQ - TIME(2))/60                                               ;

COMMENT CAL NUSSELT NUMBERS, DIST, AND ERRORS                        ;
COMMENT THIS SECTION VARIES FROM PROGRAM TO PROGRAM. FOR HEAT TRANSFER
PROGRAMS IT STANDS AS IT IS HERE.  FOR MASS TRANSFER PROGRAMS TEMP,
```

```
PRN AND KT ARE REPLACED BY CONC, SC AND RDF.  FOR TUBULAR REACTOR
PROGRAMS ONLY THE ERROR CHECKS AND LNUSR ARE RETAINED                      ;
NUSO ← PRNxTAVG/(4xKTMxZP)                                                 ;
NUSAV ← PRNxTAVG/(2xKTAVx(2-TAVG))                                         ;
NUSLN ← PRNx(-LN(1-TAVG))/(4xZP)                                           ;
LNUSR ← (3xT[2,M+1] -4xT[2,M] + T[2,M-1])/(2xDRx(1-TAVG))                  ;
ERRAD ← 4xLNUSRx(TAVG-1)xDZ/(PRNxHR[1]x(WAVG1 - WAVG))                     ;
ERRA ← 1 - (TAVG - TAVG1)/(WAVG1 - WAVG)                                   ;
ERR ← (ERRA - ERRAD)/(ERRA + ERRAD)                                        ;
AXZ ← 16xZPx(1+N1)/(PRNx(1+3xN1))                                          ;
GZ ← 3.1416 x PRN/(4xZP)                                                   ;
LAMDA ← AXZ x ALP                                                          ;

COMMENT PRINT OUT - IN ORDER TO REDUCE THE AMOUNT OF OUTPUT THE
FOLLOWING SCHEME WAS USED                                                  ;
IF WAVG ≤ ZPT THEN BEGIN
    IF ZPT < 0.0485 THEN ZPT + ZPT + 0.01                                  ;
    IF ZPT < 0.8985 AND ZPT > 0.0485 THEN ZPT ← ZPT + 0.05                 ;
    IF ZPT > 0.8985 THEN ZPT ← ZPT + 0.01                                  ;
    RAD1 ← -0.00001                                                        ;
    WRITE(FOUT,HOW3,OUT3)                                                  ;
    WRITE(FOUT,FMT4)                                                       ;
    FOR I ← 1 STEP 1 UNTIL M+1 DO BEGIN
          RAD ← R[I]                                                       ;
          IF RAD ≥ RAD1 THEN BEGIN RAD1 ← RAD1 + 0.05                      ;
              WRITE(FOUT,HOW4,OUT4)                                        ;
```

```
            END                                                   ;
        END                                                       ;
        END                                                       ;
END                                                               ;

COMMENT CHANGE DR                                                 ;
K1 ← K1 + 1                                                       ;
IF VMAX < 100 AND K1 ≥ 10 AND M > 80 THEN K1   0                  ;
IF VMAX <  50 AND K1 ≥ 10 AND M > 40 THEN K1   0                  ;
IF VMAX <  10 AND K1 ≥ 10 AND M > 20 THEN K1   0                  ;
IF K1 = 0 THEN BEGIN
     M ← M/2      ;        DR ← 0.5/M                             ;
     FOR I ← 1 STEP 1 UNTIL M+1 DO BEGIN
          R[I] ← (I-1)xDR                                         ;
          V[I] ← V[2xI-1]                                         ;
          W[2,I] ← W[2,2xI-1]                                     ;
          U[2,I] ← U[2,2xI-1]                                     ;
          T[2,I] ← T[2,2xI-1]                                     ;
END          ;          END                                       ;

COMMENT CHANGE DZ                                                 ;
IF ERR < 0.05 AND ABS(WAVG - WAVG1) < 0.005 THEN BEGIN
     DZ ← 2xDZ    ;    WRITE(FOUT,FMT5)                           ;
END                                                               ;

COMMENT REPEAT CYCLE                                              ;
IF WAVG > 0.02 THEN BEGIN PR1 ← PR2                               ;
```

```
    FOR I ← 1 STEP 1 UNTIL M + 1 DO BEGIN
            U[1,I] ← U[2,I]                                      ;
            T[1,I] ← T[2,I]                                      ;
            W[1,I] ← W[2,I]                                      ;
    END                                                          ;
    WAVG1 ← WAVG                                                 ;
    GO TO TOM                                                    ;
END                                                              ;
LEND:  END.
```

APPENDIX C

PROPERTY EQUATIONS

Although the program is valid for variable properties, all properties were not varied in every program and some were never varied at all. The heat capacity and thermal conductivity were not varied and were always equal to unity. The heat of reaction and the flow behavior index were varied from program to program but never varied within a given program.

The density was varied widely in some programs but almost always a linear variation with concentration was assumed. For gases the perfect gas assumption gives

$$\rho = \frac{(c_1 + c_2 w)}{T}$$

where

$$c_1 + c_2 = 1$$

The reaction rate constant was assumed to follow the Arrhenius relation

$$\ln K_r' = \ln K_r' + \frac{\Delta H_a'}{\mathcal{R}' T'}$$

or

$$K_r' = K_r' \exp\left(\frac{\Delta \hat{H}_a'}{\mathcal{R}' T'}\right)$$

Then

$$K_r = \frac{K_r'}{K_{ro}'}$$

$$= \exp\left(\frac{\Delta H_a'}{\mathcal{R}'T'}\left(\frac{1}{T'} - \frac{1}{T_o'}\right)\right)$$

$$= \exp\left(\frac{\Delta H_a'}{\mathcal{R}'T_o'}\left(\frac{T - 1}{T}\right)\right)$$

$$= \exp\left(E_a \frac{T - 1}{T}\right)$$

The work of Christensen and Craig (19) demonstrated that the flow consistency index also follows the Arrhenius relation. Thus for the case of the wall temperature equal to the entering bulk temperature

$$K_v = \exp\left(E_v \frac{T - 1}{T}\right)$$

where

$$E_v = \frac{\Delta \overline{H}_v'}{\mathcal{R}'T_o'}$$

For the Graetz problem the dimensionless temperature is defined differently as

$$T = \frac{T' - T_w'}{T_i' - T_w'}$$

Then K_v becomes

$$K_v = \exp\left(\frac{\Delta \overline{H}'_{va} T_i' - T_w'}{T_i' T_w'} \frac{T_w T}{1 + (T_w - 1)T}\right)$$

By defining

$$E_v = \frac{\Delta \overline{H}'_{va}}{\mathcal{R}'} \frac{T_i' - T_w'}{T_i' T_w'}$$

K_v becomes

$$K_v = \exp\left(E_v \frac{T_w T}{1 + (T_w - 1)T}\right)$$

The results of this work and those of Craig (19) indicate that

$$K_v = \exp(E_v\ T)$$

gives results to within one per cent for all values of T_w less than 1.3.

APPENDIX D

SIMPLE MODELS

The results of the simple models proposed in Chapter III of this work require a number of numerical integrations. These integrations were performed by a straightforward use of Simpson's rule. A number of cases of interest are presented in this section.

Table 4. Constant Property Comparison of "Plug Flow" and "Parabolic Flow" Models

z^*	W_m $\alpha = 0$	$\alpha = \infty$	z^*	W_m $\alpha = 0$	$\alpha = \infty$
	$n = 1.5$			$n = 1.0$	
0	1.000	1.000	0	1.000	1.000
0.2	0.644	0.684	0.2	0.670	0.704
0.4	0.415	0.490	0.4	0.449	0.514
0.6	0.267	0.359	0.6	0.302	0.383
0.8	0.172	0.267	0.8	0.202	0.289
1.0	0.111	0.200	1.0	0.135	0.219
1.2	0.071	0.151	1.2	0.097	0.167
1.4	0.046	0.155	1.4	0.061	0.129
1.6	0.030	0.088	1.6	0.041	0.100
	$n = 0.5$			$n = 0.2$	
0	1.000	1.000	0	0.000	1.000
0.2	0.716	0.736	0.2	0.767	0.777
0.4	0.514	0.558	0.4	0.586	0.615
0.6	0.368	0.429	0.6	0.449	0.487
0.8	0.264	0.332	0.8	0.344	0.384
1.0	0.189	0.258	1.0	0.263	0.306
1.2	0.135	0.201	1.2	0.202	0.244
1.4	0.097	0.158	1.4	0.154	0.196
1.6	0.069	0.124	1.6	0.118	0.157

Table 5. Average Concentration of Variable Density "Parabolic Flow" Model

z^*	n=0.2	n=0.5	n=1.0	n=1.5
		$c_1 = 1.1$		
0	1.000	1.000	1.000	1.000
0.2	0.775	0.735	0.700	0.670
0.4	0.604	0.552	0.506	0.481
0.6	0.473	0.418	0.372	0.347
0.8	0.371	0.319	0.276	0.254
1.0	0.292	0.244	0.206	0.187
1.2	0.229	0.187	0.155	0.139
1.4	0.181	0.144	0.117	0.103
1.6	0.143	0.111	0.089	0.078
		$c_1 = 0.9$		
0	1.000	1.000	1.000	1.000
0.2	0.780	0.742	0.708	0.689
0.4	0.617	0.567	0.523	0.499
0.6	0.493	0.440	0.395	0.371
0.8	0.396	0.344	0.302	0.280
1.0	0.320	0.272	0.233	0.214
1.2	0.259	0.216	0.182	0.165
1.4	0.211	0.172	0.143	0.128
1.6	0.172	0.138	0.112	0.100

Table 6. Average Concentration of Variable Density "Parabolic Flow" Model - Newtonian Fluid

	$\hat{c}_1$								
z^*	0.333	0.500	0.667	0.750	1.250	1.500	1.667	2.000	3.000
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.2	0.731	0.724	0.717	0.714	0.693	0.683	0.676	0.663	0.621
0.4	0.572	0.558	0.543	0.536	0.493	0.472	0.458	0.431	0.353
0.6	0.464	0.443	0.423	0.413	0.355	0.327	0.310	0.276	0.190
0.8	0.385	0.359	0.335	0.323	0.257	0.227	0.209	0.175	0.099
1.0	0.325	0.296	0.269	0.256	0.187	0.158	0.141	0.110	0.050
1.2	0.277	0.246	0.218	0.204	0.137	0.110	0.095	0.069	0.026
1.4	0.239	0.207	0.178	0.165	0.100	0.077	0.064	0.044	0.013
1.6	0.208	0.175	0.146	0.133	0.074	0.054	0.043	0.027	0.007

Table 7. Adiabatic "Plug Flow" Integral.

	$\Delta H = -0.1$			$\Delta H = -0.3$			$\Delta H = -0.5$		
1-w \ E_a	7	14	21	7	14	21	7	14	21
0.02	0.0201	0.0199	0.198	0.0198	0.0194	0.0190	0.0195	0.0189	0.0182
0.04	0.0403	0.0397	0.0392	0.0392	0.0376	0.0361	0.0381	0.0356	0.0334
0.06	0.0606	0.0593	0.0581	0.0581	0.0547	0.0515	0.0559	0.0506	0.0460
0.08	0.0811	0.0788	0.0767	0.0768	0.0708	0.0655	0.0728	0.0640	0.0566
0.10	0.1017	0.0982	0.0949	0.0951	0.0860	0.0782	0.0891	0.0760	0.0655
0.15	0.1541	0.1463	0.1390	0.1394	0.1205	0.1050	0.1270	0.1012	0.0823
0.20	0.2079	0.1940	0.1813	0.1822	0.1507	0.1263	0.1615	0.1209	0.0935
0.25	0.2632	0.2414	0.2219	0.2236	0.1774	0.1434	0.1933	0.1366	0.1013
0.30	0.3204	0.2888	0.2613	0.2641	0.2011	0.1574	0.2229	0.1493	0.1068
0.35	0.3798	0.3365	0.2995	0.3039	0.2225	0.1689	0.2508	0.1598	0.1107
0.40	0.4420	0.3848	0.3370	0.3433	0.2419	0.1785	0.2773	0.1686	0.1136
0.45	0.5074	0.4339	0.3739	0.3827	0.2598	0.1866	0.3028	0.1761	0.1158
0.50	0.5768	0.4844	0.4107	0.4225	0.2764	0.1935	0.3277	0.1826	0.1175
0.55	0.6511	0.5368	0.4476	0.4632	0.2921	0.1996	0.3533	0.1883	0.1189
0.60	0.7315	0.5919	0.4852	0.5052	0.3071	0.2049	0.3769	0.1935	0.1200
0.65	0.8200	0.6504	0.5240	0.5494	0.3217	0.2098	0.4021	0.1983	0.1209
0.70	0.9189	0.7139	0.5648	0.5968	0.3363	0.2143	0.4285	0.2028	0.1216
0.75	1.0325	0.7846	0.6088	0.6490	0.3512	0.2185	0.4568	0.2072	0.1223
0.80	1.1673	0.8661	0.6581	0.7085	0.3671	0.2228	0.4883	0.2116	0.1229
0.85	1.3359	0.9649	0.7160	0.7801	0.3849	0.2272	0.5255	0.2165	0.1236
0.90	1.5666	1.0961	0.7907	0.8745	0.4070	0.2324	0.5735	0.2221	0.1242
0.95	1.9492	1.3093	0.9073	1.0255	0.4399	0.2395	0.6490	0.2304	0.1251
0.98	2.4441	1.5743	1.0513	1.2156	0.4793	0.2477	0.7427	0.2399	0.1261

Table 8. Adiabatic "Parabolic Flow" Concentration

	$E_a = 7$				$E_a = 14$				$E_a = 21$			
z^* n	0.2	0.5	1.0	1.5	0.2	0.5	1.0	1.5	0.2	0.5	1.0	1.5
0.05	0.936	0.921	0.906	0.898	0.934	0.918	0.901	0.892	0.933	0.914	0.895	0.844
0.10	0.875	0.847	0.821	0.806	0.868	0.836	0.806	0.789	0.860	0.822	0.788	0.769
0.16	0.816	0.778	0.743	0.724	0.801	0.757	0.717	0.694	0.783	0.732	0.685	0.658
0.20	0.759	0.714	0.673	0.650	0.736	0.683	0.635	0.608	0.706	0.644	0.586	0.554
0.25	0.705	0.654	0.609	0.584	0.672	0.612	0.559	0.529	0.630	0.558	0.493	0.459
0.30	0.655	0.599	0.551	0.524	0.611	0.546	0.489	0.458	0.555	0.476	0.410	0.375
0.35	0.607	0.549	0.480	0.471	0.553	0.484	0.425	0.394	0.482	0.401	0.344	0.301
0.40	0.562	0.502	0.450	0.422	0.498	0.427	0.368	0.338	0.412	0.331	0.268	0.237
0.60	0.407	0.347	0.297	0.272	0.308	0.245	0.197	0.173	0.181	0.128	0.092	0.076
0.08	0.290	0.236	0.194	0.174	0.175	0.130	0.097	0.083	0.059	0.038	0.025	0.019
1.00	0.203	0.159	0.127	0.111	0.092	0.065	0.046	0.038	0.016	0.009	0.005	0.004
1.20	0.141	0.106	0.082	0.070	0.046	0.031	0.021	0.016				
1.40	0.096	0.070	0.053	0.045	0.023	0.014	0.009	0.006				
1.60	0.066	0.047	0.034	0.028	0.010	0.006	0.003	0.002				
0.05	0.933	0.915	0.896	0.885	0.923	0.896	0.872	0.859	0.911	0.874	0.839	0.815
0.10	0.861	0.824	0.791	0.772	0.822	0.772	0.719	0.687	0.770	0.767	0.592	0.547
0.15	0.786	0.736	0.691	0.645	0.712	0.629	0.554	0.513	0.557	0.426	0.320	0.268
0.20	0.711	0.651	0.596	0.565	0.582	0.478	0.393	0.349	0.252	0.142	0.080	0.057
0.25	0.637	0.569	0.508	0.475	0.436	0.329	0.249	0.211	0.005	0.001		
0.30	0.566	0.492	0.428	0.395	0.287	0.195	0.134	0.108				
0.35	0.497	0.420	0.357	0.324	0.153	0.094	0.058	0.044				
0.40	0.431	0.355	0.294	0.263	0.062	0.034	0.020	0.014				
0.60	0.217	0.161	0.121	0.104								
0.80	0.092	0.062	0.043	0.036								
1.00	0.035	0.022	0.014	0.010								

(continued)

Table 8. Adiabatic "Parabolic Flow" Concentration (Concluded)

	$E_a = 7$				$E_a = 14$				$E_a = 21$			
z^* \ n	0.2	0.5	1.0	1.5	0.2	0.5	1.0	1.5	0.2	0.5	1.0	1.5
0.05	0.929	0.905	0.882	0.870	0.905	0.869	0.830	0.802	0.869	0.780	0.730	0.672
0.10	0.841	0.796	0.754	0.729	0.753	0.655	0.565	0.518	0.513	0.336	0.208	0.164
0.15	0.745	0.683	0.622	0.588	0.516	0.380	0.276	0.227				
0.20	0.647	0.567	0.496	0.457	0.171	0.090	0.047	0.033				
0.25	0.544	0.454	0.380	0.342								
0.30	0.440	0.350	0.279	0.245								
0.35	0.341	0.258	0.197	0.168								
0.40	0.251	0.181	0.132	0.110								

Table 9. General Solution to "Plug Flow" Model $\Delta\hat{H} = 0.1$

$\Delta H = -0.1 \quad E_a = 7$

$w\,\frac{NuSc}{\alpha Pr}$	0	4	10	24
0.95	0.025	0.025	0.025	0.025
0.90	0.051	0.051	0.051	0.051
0.80	0.105	0.105	0.106	0.107
0.60	0.226	0.229	0.234	0.241
0.40	0.378	0.394	0.411	0.432
0.20	0.611	0.669	0.722	0.766
0.10	0.828	0.955	1.048	1.106

$\Delta H = -0.1 \quad E_a = 14$

$w\,\frac{NuSc}{\alpha Pr}$	0	4	10	16	24
0.95	0.025	0.025	0.025	0.025	0.025
0.90	0.049	0.049	0.050	0.050	0.050
0.80	0.098	0.099	0.100	0.101	0.101
0.60	0.196	0.202	0.210	0.217	0.224
0.40	0.305	0.327	0.357	0.379	0.400
0.20	0.452	0.575	0.620	0.678	0.718
0.10	0.577	0.730	0.915	1.001	1.051

$\Delta H = -0.1 \quad E_a = 21$

$w\,\frac{NuSc}{\alpha Pr}$	0	4	10	16	24
0.95	0.024	0.024	0.025	0.025	0.025
0.90	0.048	0.048	0.048	0.048	0.049
0.80	0.092	0.093	0.094	0.095	0.097
0.60	0.172	0.179	0.188	0.197	0.207
0.40	0.250	0.271	0.305	0.335	0.366
0.20	0.342	0.404	0.512	0.600	0.666
0.10	0.414	0.534	0.763	0.906	0.992

Table 10. General Solution to "Plug Flow" Model
$\Delta\hat{H} = -0.3$

$\Delta\hat{H} = -0.3 \quad E_a = 7$

w \ $\frac{NuSc}{\alpha Pr}$	0	4	10	16	24
0.95	0.025	0.025	0.025	0.025	0.025
0.90	0.048	0.048	0.049	0.049	0.049
0.80	0.094	0.095	0.096	0.097	0.098
0.60	0.182	0.188	0.197	0.204	0.213
0.40	0.275	0.295	0.326	0.352	0.378
0.20	0.398	0.459	0.557	0.630	0.685
0.10	0.502	0.627	0.827	0.942	1.012

$\Delta\hat{H} = -0.3 \quad E_a = 14$

w \ $\frac{NuSc}{\alpha Pr}$	0	10	16	24
0.95	0.023	0.023	0.023	0.024
0.90	0.044	0.044	0.045	0.045
0.80	0.077	0.081	0.082	0.085
0.60	0.127	0.142	0.152	0.164
0.40	0.165	0.201	0.230	0.272
0.20	0.201	0.283	0.373	0.504
0.10	0.226	0.373	0.585	0.802

$\Delta\hat{H} = -0.3 \quad E_a = 21$

w \ $\frac{NuSc}{\alpha Pr}$	0	10	16	24	30	40
0.95	0.022	0.022	0.022	0.022	0.023	0.023
0.90	0.040	0.040	0.041	0.041	0.042	0.042
0.80	0.065	0.068	0.070	0.072	0.074	0.077
0.60	0.093	0.104	0.111	0.122	0.131	0.147
0.40	0.108	0.127	0.142	0.170	0.198	0.250
0.20	0.119	0.147	0.175	0.250	0.356	0.354
0.10	0.125	0.159	0.202	0.401	0.632	0.813

Table 11. General Solution to "Plug Flow" Model $\Delta\hat{H} = -0.5$

			$\Delta\hat{H} = -0.5$	$E_a = 7$	
w \ $\frac{NuSc}{\alpha Pr}$	0	4	10	16	24
0.95	0.024	0.024	0.024	0.024	0.024
0.90	0.046	0.046	0.046	0.046	0.047
0.80	0.085	0.086	0.087	0.089	0.091
0.60	0.151	0.157	0.166	0.175	0.177
0.40	0.213	0.230	0.258	0.287	0.323
0.20	0.289	0.329	0.410	0.504	0.596
0.10	0.349	0.421	0.598	0.780	0.909

			$\Delta\hat{H} = -0.5$	$E_a = 14$		
w \ $\frac{NuSc}{\alpha Pr}$	0	10	16	24	30	40
0.95	0.022	0.022	0.022	0.022	0.022	0.022
0.90	0.039	0.040	0.040	0.040	0.041	0.041
0.80	0.063	0.066	0.068	0.070	0.072	0.075
0.60	0.090	0.100	0.107	0.117	0.125	0.140
0.40	0.106	0.123	0.136	0.160	0.185	0.234
0.20	0.118	0.143	0.167	0.228	0.318	0.474
0.10	0.125	0.157	0.194	0.344	0.575	0.780

			$\Delta\hat{H} = -0.5$	$E_a = 21$	
w \ $\frac{NuSc}{\alpha Pr}$	0	40	60	80	100
0.95	0.020	0.020	0.021	0.021	0.022
0.90	0.034	0.036	0.038	0.039	0.058
0.80	0.049	0.059	0.065	0.070	0.076
0.60	0.060	0.087	0.112	0.142	0.166
0.40	0.064	0.104	0.178	0.272	0.319
0.20	0.066	0.118	0.406	0.558	0.620
0.10	0.067	0.128	0.713	0.877	0.945

APPENDIX E

ANALYTICAL SOLUTIONS

The eigenvalues, expansion coefficients and norms required for use with the analytical solutions developed in Chapter IV of this work are presented. For the heat-transfer problem, Nusselt numbers and mean temperatures are presented as functions of ξ.

Table 12. Eigenvalues, Expansion Coefficients and Norms for the Tubular Reactor Problem

		N = 1.5			N = 1.0		
α	i	λ_i	C_i	$\overline{N}_i$	λ_i	C_i	$\overline{N}_i$
0.25	1	0.53556	1.06580	0.19978	0.48984	1.0608	0.22187
	2	7.44997	-0.08284	0.04391	6.9224	-0.07832	0.04755
	3	23.1106	0.02564	0.02496	21.467	0.02569	0.02694
	4	47.4236	-0.01116	0.01746	44.043	-0.01198	0.01883
	5	80.3955	0.00882	0.01346	74.632	0.00851	0.01449
	6				113.152	-0.00106	0.01184
0.50	1	1.0518	1.12866	0.17739	0.9605	1.1180	0.19904
	2	8.0108	-0.16600	0.04393	7.4290	-0.1531	0.04756
	3	23.6795	0.05450	0.02496	21.971	0.0514	0.02695
	4	47.971	-0.02432	0.01746	44.545	-0.0247	0.01883
	5	80.952	0.01693	0.01346	75.136	0.0161	0.01450
	6				113.022	-0.0138	0.01184
1.00	1	2.0364	1.24472	0.14346	1.8504	1.2202	0.16497
	2	9.1394	-0.31187	0.04394	8.4493	-0.2906	0.04757
	3	24.7966	0.10945	0.02497	22.985	0.1031	0.02696
	4	49.0696	-0.05098	0.01747	45.555	-0.0505	0.01883
	5	82.0520	0.03284	0.01346	76.135	0.0310	0.01450
	6				115.051	-0.0277	0.01183
2.50	1	4.5724	1.47025	0.09504	4.2146	1.4340	0.11211
	2	12.4919	-0.64238	0.04315	11.492	-0.6064	0.04689
	3	28.1945	0.27194	0.02499	26.063	0.2560	0.02698
	4	52.3889	-0.13313	0.01749	48.615	-0.1304	0.01885
	5	85.4242	0.08428	0.01347	79.186	0.0790	0.01451
	6				118.051	-0.0566	0.01183
5.00	1	7.9335	1.60451	0.07498	7.6012	1.5986	0.08042
	2	17.6426	-0.95019	0.03977	16.239	-0.9118	0.04366
	3	33.8385	0.50706	0.02478	31.190	0.4810	0.02680
	4	58.1881	-0.28253	0.01749	53.778	-0.2654	0.01885
	5	91.1001	0.17383	0.01348	84.325	0.1628	0.01182
25.0	1	31.7719	1.81807	0.03472	30.523	1.8047	0.04119
	2	48.1876	-1.47619	0.02615	45.109	-1.4408	0.02910
	3	70.2374	1.15872	0.01985	65.142	1.1362	0.02178
	4	98.7445	-0.91028	0.01569	91.243	-0.8870	0.01710
	5				123.76	0.6874	0.01396

(continued)

Table 12. Eigenvalues, Expansion Coefficients and Norms for the Tubular Reactor Problem (Concluded)

		N = 0.5			N = 0.2		
α	i	λ_i	C_i	$\bar{N}_i$	λ_i	C_i	$\bar{N}_i$
0.25	1	0.40170	1.0458	0.27410	0.25000	0.9984	0.37617
	2	6.0251	-0.06398	0.05477	5.07973	-0.0411	0.06397
	3	18.693	0.02203	0.03094	15.8796	0.0154	0.03627
	4	38.362	-0.01038	0.021606	32.6527	-0.0076	0.02532
	5	65.049	0.00886	0.016623	55.3896	0.0054	0.01947
	6	98.6139	-0.00091	0.013539	84.830	-0.0018	0.01584
0.50	1	0.80958	1.0950	0.24941	0.50000	0.9969	0.37735
	2	6.4454	-0.1256	0.05478	5.41407	-0.0814	0.06394
	3	19.112	0.0440	0.03099	16.2136	0.0309	0.03627
	4	38.780	-0.0212	0.02161	32.9865	-0.0155	0.02532
	5	65.470	0.0155	0.01662	55.7234	0.0101	0.01947
	6	99.020	-0.0046	0.01354	84.4196	-0.0514	0.01584
1.00	1	1.5794	1.1804	0.21283	1.0010	1.0004	0.37469
	2	7.2907	-0.2408	0.05478	6.0843	-0.1582	0.06397
	3	19.953	0.0882	0.03095	16.8830	0.0616	0.03627
	4	39.619	-0.0433	0.02161	33.6551	-0.0314	0.02532
	5	66.314	0.0290	0.01662	56.3917	0.0198	0.01947
	6	99.872	-0.0139	0.01354	85.0872	-0.0115	0.01584
2.50	1	3.6948	1.3719	0.15060	3.1552	1.2584	0.23088
	2	9.8221	-0.5198	0.05431	8.0967	-0.3604	0.06378
	3	22.499	0.2189	0.03097	18.8996	0.1529	0.03628
	4	42.154	-0.1113	0.02162	35.6680	-0.0880	0.02533
	5	68.846	0.0701	0.01663	58.4014	0.0492	0.01947
5.00	1	6.8351	1.5392	0.10921	6.0596	1.4144	0.17376
	2	14.001	-0.8299	0.05119	11.4109	-0.6087	0.06255
	3	26.759	0.41754	0.03087	22.2733	0.2963	0.03625
	4	46.423	-0.2266	0.02163	39.0404	-0.1616	0.02533
	5	73.052	0.1388	0.01664	61.7668	0.0998	0.01948
25.0	1	28.637	1.7452	0.05862	27.0684	1.6525	0.09670
	2	40.037	-1.3493	0.03588	35.0029	-1.1662	0.04787
	3	56.611	1.0713	0.02602	47.8189	0.8995	0.03284
	4	78.616	-0.8225	0.02013	65.5926	-0.6641	0.02454

Table 13. Eigenvalues, Expansion Coefficients and Norms for the Graetz Problem

i	λ_i	C_i	$\overline{N}_i$	λ_i	C_i	$\overline{N}_i$
	N = 1.5			N = 1.0		
1	1.94401	1.4698	0.0864	1.8284	1.4764	0.09394
2	12.0080	-0.7960	0.0347	11.1523	-0.8060	0.03755
3	30.6833	0.5803	0.0217	28.4804	0.5887	0.02345
4	57.9810	-0.4669	0.0158	53.8097	-0.4743	0.01708
5				87.1380	0.4048	0.01345
	N = 0.8			N = 0.6		
1	1.7442	1.4777	0.0996	1.6763	1.4865	0.10565
2	9.7819	-0.7271	0.0419	10.1258	-0.8286	0.04157
3	22.3289	0.2290	0.0270	25.7695	0.6062	0.02597
4				48.6487	-0.4889	0.01890
	N = 0.5			N = 0.4		
1	1.6455	1.4944	0.1091	1.6127	1.5051	0.11310
2	9.7728	-0.8381	0.0431	9.3650	-0.8510	0.04504
3	24.8774	0.6145	0.02692	23.8499	0.6256	0.02809
4	46.9481	-0.4960	0.0196	44.9958	-0.5052	0.02044
5	75.9955	0.4232	0.0154	74.8234	0.4311	0.01608
6	112.008	-0.3672	0.0127			
	N = 0.2					
1	1.4998	1.5343	0.1262			
2	8.4234	-0.9083	0.0504			
3	21.3116	0.6692	0.0315			
4	40.1496	-0.5417	0.0229			
5	64.9377	0.4624	0.0180			
6	95.6759	-0.4033	0.0149			

Table 14. Nusselt Numbers for the Graetz Problem

ξ	Nu_L	Nu_o	Nu_{AM}	Nu_{LN}	T_m	Nu_L	Nu_o	Nu_{AM}	Nu_{LN}	T_m
			N = 1.5					N - 1.0		
0.1	4.28	5.35	6.27	6.34	0.294	4.64	5.68	6.62	6.68	0.284
0.2	3.83	3.95	5.04	5.18	0.434	4.00	4.21	5.34	5.47	0.421
0.3	3.66	3.27	4.47	4.69	0.539	3.79	3.49	4.72	4.94	0.523
0.4	3.60	2.83	4.10	4.42	0.622	3.71	3.02	4.33	4.64	0.605
0.5	3.57	2.50	3.82	4.25	0.689	3.68	2.69	4.04	4.45	0.671
1.0	3.56	1.60	2.87	3.89	0.882	3.65	1.74	3.07	4.06	0.868
1.5	3.56	1.16	2.22	3.77	0.956	3.65	1.26	2.40	3.92	0.947
			N = 0.8					N = 0.6		
0.1	-	-	-	-	-	5.08	6.18	7.14	7.20	0.270
0.2	4.37	4.24	5.30	5.42	0.401	4.40	4.60	5.76	5.88	0.402
0.3	3.95	3.58	4.79	4.99	0.507	4.14	3.83	5.09	5.30	0.501
0.4	3.80	3.12	4.42	4.71	0.589	4.03	3.32	4.67	4.96	0.580
0.5	3.74	2.78	4.13	4.52	0.656	3.98	2.95	4.36	4.75	0.646
1.0	3.69	1.88	3.25	4.13	0.843	3.95	1.99	3.36	4.29	0.847
1.5	3.69	1.33	2.50	3.97	0.940	3.95	1.42	2.67	4.14	0.934
			N = 0.5					N = 0.4		
0.1	5.20	6.48	4.50	7.56	0.270	5.37	6.90	7.98	8.05	0.271
0.2	4.44	4.81	6.02	6.15	0.401	4.53	5.10	6.38	6.52	0.401
0.3	4.16	3.99	5.32	5.53	0.499	4.22	4.22	5.62	5.84	0.497
0.4	4.04	3.47	4.87	5.17	0.578	4.09	3.66	5.14	5.45	0.575
0.5	3.99	3.08	4.54	4.94	0.643	4.02	3.26	4.79	5.19	0.640
1.0	3.95	2.02	3.50	4.45	0.843	3.97	2.14	3.68	4.66	0.839
1.5	3.95	1.49	2.79	4.28	0.931	3.97	1.58	2.94	4.47	0.928
			N = 0.2							
0.1	6.27	7.97	9.19	9.26	0.266					
0.2	5.26	5.88	7.32	7.47	0.392					
0.3	4.87	4.86	6.42	6.66	0.486					
0.4	4.68	4.21	5.85	6.18	0.561					
0.5	4.59	3.75	5.44	5.87	0.624					
1.0	4.50	2.47	4.20	5.20	0.823					
1.5	4.50	1.83	3.38	4.97	0.917					

APPENDIX F

TUBULAR REACTOR RESULTS

Correlation of the numerical solutions for the tubular reactor problem appear in this section. The correlating group $\alpha Pr/Sc$ has been discussed in the main body of this work. The Schmidt number was usually taken to be 1 for gases, 1,000 for liquids, and 10,000 for non-Newtonian fluids. The Prandtl number was usually taken to be 1 for gases, 10 for liquids and 100 for non-Newtonian fluids.

Table 15. Constant Property Solution - Newtonian Fluids Concentration Profiles - Entrance Velocity Profile: Parabolic

α/Sc	W_m	Z^*	r = 0 W	r = 0.1 W	r = 0.2 W	r = 0.3 W	r = 0.4 W	r=0.5 W
0.25	0.898	0.055	0.935	0.931	0.916	0.892	0.866	0.853
	0.799	0.115	0.845	0.838	0.818	0.791	0.765	0.753
	0.597	0.264	0.633	0.628	0.611	0.590	0.570	0.561
	0.399	0.469	0.424	0.420	0.409	0.395	0.381	0.375
	0.198	0.825	0.210	0.208	0.203	0.196	0.189	0.186
	0.089	1.227	0.095	0.094	0.091	0.088	0.085	0.084
2.50	0.897	0.056	0.945	0.942	0.933	0.908	0.828	0.748
	0.800	0.118	0.886	0.881	0.861	0.804	0.688	0.610
	0.597	0.279	0.741	0.727	0.677	0.578	0.459	0.399
	0.398	0.509	0.550	0.528	0.464	0.370	0.281	0.242
	0.198	0.914	0.298	0.280	0.235	0.178	0.131	0.112
	0.089	1.375	0.139	0.130	0.107	0.079	0.058	0.049
25.00	0.898	0.057	0.944	0.942	0.934	0.914	0.845	0.600
	0.800	0.121	0.885	0.881	0.865	0.825	0.689	0.401
	0.600	0.291	0.746	0.737	0.704	0.624	0.401	0.183
	0.398	0.553	0.571	0.557	0.509	0.397	0.191	0.074
	0.199	1.027	0.349	0.332	0.278	0.175	0.062	0.021
	0.088	1.616	0.185	0.171	0.128	0.066	0.019	0.006
140.00	0.896	0.058	0.943	0.941	0.933	0.913	0.849	0.513
	0.796	0.125	0.882	0.877	0.861	0.821	0.700	0.261
	0.600	0.296	0.743	0.734	0.702	0.627	0.424	0.065
	0.398	0.564	0.567	0.553	0.508	0.408	0.192	0.014
	0.201	1.048	0.347	0.332	0.282	0.187	0.050	0.002
	0.084	1.693	0.178	0.165	0.127	0.064	0.010	0.000

Table 16. Constant Property Solution
Concentration Profiles
Entrance Velocity Profile: Parabolic

n	α/Sc	W_m	Z^*	r=0	r=0.2	r=0.3	r=0.4	r=0.5
1.5	10	0.794	0.116	0.890	0.862	0.816	0.686	0.150
		0.601	0.274	0.759	0.703	0.618	0.410	0.000
		0.397	0.530	0.587	0.506	0.394	0.178	0.000
		0.196	1.005	0.363	0.274	0.170	0.037	0.000
		0.083	1.620	0.194	0.123	0.056	0.004	0.000
1.0	10	0.800	0.123	0.884	0.864	0.825	0.709	0.195
		0.597	0.302	0.739	0.697	0.623	0.430	0.000
		0.399	0.568	0.565	0.507	0.409	0.203	0.000
		0.200	1.060	0.344	0.281	0.188	0.050	0.000
		0.088	1.674	0.184	0.133	0.070	0.008	0.000
0.5	10	0.797	0.147	0.863	0.854	0.828	0.738	0.272
		0.597	0.350	0.704	0.687	0.639	0.486	0.186
		0.396	0.657	0.517	0.494	0.430	0.257	0.000
		0.196	1.210	0.296	0.272	0.211	0.081	0.000
		0.090	1.849	0.154	0.135	0.091	0.021	0.000
0.2	10	0.796	0.180	0.835	0.834	0.827	0.782	0.443
		0.596	0.418	0.657	0.656	0.644	0.566	0.133
		0.398	0.764	0.464	0.463	0.447	0.353	0.006
		0.198	1.378	0.249	0.248	0.233	0.151	0.000
		0.089	2.085	0.121	0.120	0.109	0.056	0.000
1.5	1.0	0.796	0.115	0.891	0.863	0.817	0.688	0.156
		0.600	0.276	0.758	0.702	0.617	0.409	0.000
		0.398	0.529	0.588	0.507	0.395	0.179	0.000
		0.200	0.990	0.369	0.280	0.175	0.039	0.000
		0.087	1.586	0.200	0.128	0.060	0.005	0.000
0.2	1.0	0.798	0.178	0.836	0.836	0.829	0.785	0.448
		0.601	0.412	0.662	0.661	0.648	0.571	0.139
		0.396	0.768	0.463	0.461	0.445	0.350	0.007
		0.200	1.370	0.252	0.250	0.235	0.153	0.000
		0.090	2.083	0.122	0.121	0.109	0.056	0.000

Table 17. Concentration and Velocity Profiles for Gases
Entrance Velocity Profile: Uniform

			r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5
α/Sc	W_m	Z^*	w	u	w	u	w	u	w	u	w
0.1	0.895	0.052	0.935	1.99	0.919	1.67	0.897	1.28	0.872	0.723	0.859
	0.797	0.111	0.848	2.00	0.822	1.68	0.795	1.28	0.769	0.720	0.757
	0.599	0.256	0.641	2.00	0.618	1.68	0.596	1.28	0.576	0.720	0.567
	0.401	0.459	0.429	2.00	0.414	1.68	0.399	1.28	0.385	0.720	0.379
	0.199	0.813	0.213	2.00	0.205	1.68	0.198	1.28	0.191	0.720	0.188
	0.090	1.212	0.096	2.00	0.093	1.68	0.090	1.28	0.087	0.720	0.085
0.5	0.901	0.049	0.930	1.66	0.921	1.57	0.906	1.33	0.886	0.807	0.874
	0.791	0.115	0.836	1.88	0.815	1.64	0.791	1.30	0.767	0.745	0.755
	0.589	0.262	0.631	1.99	0.608	1.68	0.587	1.28	0.567	0.722	0.558
	0.395	0.459	0.423	2.00	0.408	1.68	0.393	1.28	0.379	0.720	0.372
	0.201	0.786	0.216	2.00	0.208	1.68	0.200	1.28	0.193	0.720	0.189
	0.089	1.179	0.096	2.00	0.092	1.68	0.089	1.28	0.086	0.720	0.084
1.0	0.894	0.053	0.922	1.49	0.922	1.46	0.916	1.34	0.869	0.874	0.795
	0.793	0.114	0.853	1.70	0.846	1.58	0.814	1.32	0.722	0.794	0.647
	0.601	0.262	0.718	1.91	0.677	1.65	0.597	1.29	0.486	0.740	0.424
	0.394	0.491	0.536	1.98	0.462	1.67	0.373	1.28	0.284	0.723	0.243
	0.181	0.917	0.278	2.00	0.218	1.68	0.163	1.28	0.117	0.720	0.098
	0.088	1.310	0.141	2.00	0.107	1.68	0.078	1.28	0.055	0.720	0.046
5.0	0.899	0.050	0.917	1.23	0.917	1.23	0.916	1.22	0.899	1.088	0.842
	0.796	0.111	0.831	1.33	0.831	1.33	0.821	1.30	0.769	0.993	0.697
	0.589	0.265	0.665	1.49	0.650	1.47	0.605	1.34	0.518	0.874	0.456
	0.383	0.490	0.482	1.65	0.442	1.57	0.380	1.33	0.303	0.806	0.262
	0.177	0.900	0.254	1.82	0.212	1.63	0.166	1.30	0.124	0.759	0.104
	0.081	1.309	0.125	1.91	0.098	1.65	0.073	1.29	0.052	0.739	0.044

Table 18. Concentration and Velocity Profiles for Non-Newtonian Fluids
Entrance Velocity Profile: Uniform

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5
n	α/Sc	W_m	Z^*	w	u	w	u	w	u	w	u	w
1.5	1	0.924	0.037	0.946	1.74	0.945	1.65	0.938	1.32	0.900	0.754	0.655
		0.817	0.097	0.882	2.08	0.871	1.70	0.842	1.27	0.739	0.699	0.260
		0.610	0.253	0.748	2.19	0.707	1.72	0.633	1.26	0.434	0.684	0.000
		0.397	0.506	0.574	2.19	0.504	1.72	0.398	1.26	0.180	0.684	0.000
		0.195	0.953	0.358	2.20	0.275	1.72	0.171	1.26	0.035	0.684	0.000
		0.078	1.549	0.188	2.20	0.119	1.72	0.053	1.26	0.000	0.684	0.000
0.5	1	0.941	0.037	0.944	1.16	0.944	1.16	0.944	1.16	0.942	1.090	0.908
		0.781	0.147	0.810	1.35	0.809	1.31	0.805	1.27	0.775	0.984	0.501
		0.626	0.295	0.674	1.47	0.670	1.43	0.656	1.30	0.578	0.905	0.116
		0.417	0.550	0.501	1.57	0.491	1.50	0.460	1.31	0.332	0.854	0.000
		0.191	1.101	0.270	1.64	0.255	1.54	0.211	1.31	0.093	0.822	0.000
		0.077	1.730	0.134	1.66	0.120	1.55	0.085	1.31	0.020	0.813	0.000
0.2	1	0.935	0.052	0.934	1.03	0.934	1.03	0.934	1.03	0.934	1.02	0.927
		0.779	0.184	0.789	1.12	0.789	1.12	0.789	1.12	0.784	1.07	0.719
		0.605	0.368	0.631	1.19	0.631	1.19	0.629	1.18	0.612	1.06	0.429
		0.398	0.688	0.435	1.25	0.434	1.25	0.429	1.22	0.391	1.04	1.118
		0.185	1.278	0.222	1.30	0.221	1.30	0.214	1.25	0.163	1.01	0.000
		0.085	1.867	0.114	1.32	0.113	1.31	0.106	1.26	0.066	0.994	0.000
1.5	10	0.926	0.037	0.933	1.29	0.933	1.29	0.933	1.29	0.925	1.04	0.773
		0.809	0.100	0.840	1.43	0.840	1.43	0.838	1.37	0.787	0.893	0.401
		0.612	0.239	0.681	1.61	0.680	1.59	0.659	1.35	0.520	0.791	0.038
		0.393	0.481	0.492	1.83	0.483	1.67	0.426	1.31	0.234	0.734	0.000
		0.198	0.891	0.302	2.06	0.272	1.70	0.196	1.27	0.054	0.701	0.000
		0.087	1.412	0.169	2.16	0.131	1.72	0.071	1.26	0.007	0.688	0.000

(continued)

Table 18. Concentration and Velocity Profiles for Non-Newtonian Fluids
Entrance Velocity Profile: Uniform. (Concluded)

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5
n	$\alpha/S\dot{c}$	W_m	Z^*	w	u	w	u	w	u	w	u	w
0.5	10	0.931	0.043	0.931	1.05	0.931	1.05	0.931	1.05	0.931	1.05	0.928
		0.827	0.113	0.836	1.07	0.830	1.07	0.830	1.07	0.829	1.06	0.804
		0.612	0.292	0.625	1.14	0.625	1.14	0.625	1.14	0.620	1.09	0.489
		0.406	0.537	0.431	1.21	0.430	1.21	0.429	1.16	0.411	1.08	0.171
		0.198	0.980	0.227	1.29	0.226	1.28	0.222	1.24	0.186	1.03	0.000
		0.090	1.471	0.113	1.36	0.112	1.34	0.106	1.27	0.070	0.985	0.000
0.2	10	0.929	0.054	0.930	1.01	0.930	1.01	0.930	1.01	0.930	1.01	0.929
		0.828	0.143	0.827	1.02	0.827	1.07	0.827	1.02	0.826	1.04	0.822
		0.610	0.366	0.614	1.05	0.614	1.05	0.614	1.05	0.613	1.03	0.592
		0.400	0.673	0.408	1.07	0.408	1.07	0.407	1.07	0.405	1.05	0.359
		0.203	1.164	0.212	1.10	0.212	1.10	0.211	1.09	0.206	1.06	0.140
		0.092	1.717	0.100	1.13	0.100	1.12	0.899	1.12	0.094	1.06	0.036
0.5	0.5	0.885	0.074	0.900	1.36	0.900	1.34	0.898	1.27	0.881	0.988	0.714
		0.784	0.147	0.822	1.48	0.820	1.43	0.811	1.30	0.761	0.907	0.422
		0.620	0.295	0.694	1.59	0.687	1.51	0.662	1.31	0.556	0.850	0.104
		0.402	0.590	0.502	1.65	0.487	1.55	0.441	1.31	0.288	0.822	0.000
		0.197	1.101	0.288	1.67	0.268	1.57	0.215	1.31	0.088	0.814	0.000
		0.081	1.730	0.144	1.67	0.127	1.56	0.087	1.31	0.019	0.813	0.000
0.2	0.5	0.833	0.092	0.889	1.08	0.889	1.08	0.888	1.08	0.886	1.030	0.848
		0.781	0.184	0.795	1.20	0.795	1.19	0.794	1.18	0.783	1.021	0.659
		0.610	0.398	0.609	1.26	0.609	1.25	0.603	1.22	0.566	1.001	0.267
		0.402	0.718	0.451	1.30	0.450	1.30	0.441	1.25	0.382	0.998	0.063
		0.191	1.278	0.235	1.32	0.234	1.32	0.223	1.26	0.157	0.983	0.000
		0.092	1.868	0.122	1.33	0.121	1.33	0.112	1.27	0.063	0.980	0.000

Table 19. Effect of α/Sc and Variable Density Limit as $\theta \to 0$.

α/Sc	$\frac{\rho_{out}}{\rho_o} = 0.5$		$\frac{\rho_{out}}{\rho_o} = 0.9$		$\frac{\rho_{out}}{\rho_o} = 1.1$		$\frac{\rho_{out}}{\rho_o} = 1.5$	
	W	Z^*	W	Z^*	W	Z^*	W	Z^*
0.50	0.901	0.055	0.900	0.055	0.899	0.053	0.903	0.051
	0.795	0.125	0.793	0.121	0.799	0.120	0.793	0.113
	0.590	0.305	0.592	0.268	0.596	0.251	0.594	0.240
	0.394	0.584	0.393	0.484	0.391	0.454	0.392	0.408
	0.190	1.157	0.188	0.868	0.187	0.797	0.189	0.679
	0.090	1.812	0.093	1.298	0.089	1.157	0.088	0.940
1.00	0.902	0.055	0.900	0.055	0.899	0.055	0.899	0.053
	0.790	0.133	0.792	0.125	0.792	0.121	0.791	0.114
	0.588	0.330	0.590	0.259	0.591	0.268	0.593	0.244
	0.394	0.641	0.391	0.539	0.389	0.473	0.388	0.416
	0.189	1.313	0.192	0.981	0.189	0.825	0.193	0.678
	0.090	2.066	0.095	1.423	0.089	1.186	0.093	0.940
5.00	0.905	0.054	0.902	0.054	0.901	0.054	1.898	0.054
	0.793	0.131	0.791	0.125	0.794	0.120	0.793	0.115
	0.594	0.320	0.593	0.289	0.590	0.279	0.593	0.253
	0.391	0.637	0.389	0.540	0.389	0.504	0.389	0.448
	0.194	1.252	0.194	0.970	0.193	0.883	0.190	0.765
	0.092	1.989	0.094	1.421	0.093	1.272	0.087	1.093
10.00	0.898	0.059	0.904	0.054	0.899	0.054	0.900	0.054
	0.791	0.136	0.789	0.131	0.790	0.125	0.790	0.120
	0.590	0.340	0.593	0.305	0.591	0.294	0.590	0.274
	0.391	0.678	0.391	0.581	0.390	0.559	0.391	0.499
	0.193	1.375	0.192	1.093	0.191	1.021	0.190	0.909
	0.095	2.194	0.093	1.646	0.089	1.591	0.090	1.339

Table 20. Effect of α/Sc and Variable Density with Free Convection Effects.

Free Convection Parameter, Fc

$$Fc = \left(\frac{\rho_{out}}{\rho_o}\right)^2 \left(\frac{\rho_{out} - \rho_o}{\rho_{out} + \rho_o}\right) \theta$$

$\alpha/Sc = 1$

	Fc			
W_m	-315	-180	450	4500
0.8	1.000	1.000	1.000	1.000
0.6	1.043	1.025	0.990	0.969
0.4	1.068	1.040	0.960	0.948
0.2	1.076	1.055	0.920	0.913
0.1	1.098	1.085	0.943	0.906

$\alpha/Sc = 2.5$

	Fc				
W_m	-450	-45	450	1125	4500
0.8	1.000	1.000	1.000	1.000	1.000
0.6	1.030	1.000	0.991	0.979	0.980
0.4	1.068	1.011	0.952	0.930	0.930
0.2	1.101	1.017	0.917	0.890	0.890
0.1	1.113	1.019	0.915	0.838	0.865

$\alpha/Sc = 5$

	Fc				
W_m	-450	-85	85	425	4500
0.8	1.000	1.000	1.000	1.000	1.000
0.6	1.035	1.018	0.990	0.983	0.968
0.4	1.069	1.021	0.983	0.954	0.932
0.2	1.147	1.069	0.973	0.889	0.894
0.1	1.159	1.109	0.973	0.845	0.870

$\alpha/Sc = 10$

	F_c		
W_m	-85	425	835
0.8	1.000	1.000	1.000
0.6	1.021	0.963	0.925
0.4	1.069	0.927	0.887
0.2	1.114	0.870	0.846
0.1	1.156	0.835	0.828

Table 21. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 7$

$\frac{\alpha Pr}{Sc}$	W_m	T_m	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
0.25	0.800	1.013	0.111	0.845	1.021	0.819	1.017	0.792	1.013	0.766	1.006	0.754	1.000
	0.597	1.018	0.250	0.633	1.031	0.611	1.024	0.590	1.016	0.570	1.008	0.562	1.000
	0.396	1.015	0.442	0.420	1.026	0.406	1.020	0.392	1.013	0.379	1.006	0.373	1.000
	0.200	1.008	0.770	0.213	1.014	0.205	1.011	0.198	1.007	0.190	1.003	0.188	1.000
2.5	0.798	1.018	0.111	0.888	1.013	0.862	1.016	0.801	1.023	0.683	1.019	0.604	1.000
	0.598	1.033	0.247	0.748	1.029	0.679	1.039	0.576	1.040	0.458	1.023	0.399	1.000
	0.400	1.046	0.426	0.554	1.058	0.463	1.060	0.370	1.048	0.284	1.024	0.245	1.000
	0.201	1.055	0.713	0.293	1.088	0.235	1.072	0.183	1.051	0.138	1.024	0.118	1.000
10.0	0.798	1.019	0.115	0.887	1.011	0.866	1.013	0.825	1.019	0.681	1.033	0.402	1.000
	0.598	1.037	0.258	0.754	1.025	0.709	1.031	0.622	1.044	0.397	1.046	0.188	1.000
	0.397	1.053	0.458	0.585	1.044	0.516	1.054	0.393	1.067	0.195	1.047	0.080	1.000
	0.198	1.067	0.786	0.361	1.070	0.278	1.081	0.173	1.078	0.068	1.043	0.025	1.000
25.0	0.798	1.019	0.115	0.887	1.011	0.866	1.013	0.825	1.018	0.682	1.035	0.396	1.000
	0.600	1.038	0.256	0.756	1.025	0.713	1.029	0.628	1.040	0.395	1.057	0.182	1.000
	0.399	1.055	0.454	0.588	1.042	0.522	1.050	0.400	1.065	0.188	1.063	0.075	1.000
	0.198	1.072	0.780	0.366	1.065	0.286	1.076	0.173	1.086	0.062	1.060	0.022	1.000

Table 22. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 7$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_m	T_m	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.794	1.041	0.106	0.839	1.064	0.813	1.054	0.787	1.039	0.762	1.019	0.750	1.000
	0.599	1.057	0.221	0.632	1.096	0.613	1.075	0.593	1.052	0.515	1.052	0.566	1.000
	0.397	1.050	0.385	0.419	1.087	0.406	1.067	0.393	1.046	0.381	1.022	0.374	1.000
	0.198	1.027	0.680	0.210	1.047	0.203	1.036	0.196	1.024	0.190	1.011	0.187	1.000
2.5	0.797	1.054	0.102	0.890	1.035	0.863	1.047	0.801	1.069	0.684	1.060	0.607	1.000
	0.595	1.101	0.210	0.754	1.083	0.682	1.114	0.577	1.124	0.461	1.079	0.405	1.000
	0.396	1.143	0.335	0.562	1.162	0.463	1.181	0.371	1.158	0.291	1.088	0.255	1.000
	0.194	1.175	0.520	0.280	1.264	0.225	1.233	0.182	1.175	0.145	1.089	0.127	1.000
25.0	0.795	1.059	0.102	0.893	1.032	0.872	1.039	0.829	1.052	0.672	1.108	0.382	1.000
	0.598	1.114	0.204	0.775	1.068	0.730	1.082	0.639	1.114	0.370	1.186	0.175	1.000
	0.392	1.169	0.332	0.618	1.117	0.544	1.141	0.401	1.193	0.161	1.220	0.072	1.000
	0.196	1.219	0.508	0.403	1.183	0.307	1.216	0.167	1.264	0.052	1.221	0.023	1.000

Table 23. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.5$ $E_a = 7$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.798	1.012	0.108	0.802	1.021	0.800	1.016	0.797	1.011	0.794	1.005	0.793	1.000
	0.594	1.009	0.249	0.598	1.016	0.596	1.012	0.594	1.008	0.592	1.004	0.591	1.000
	0.396	1.006	0.446	0.399	1.010	0.397	1.008	0.396	1.005	0.394	1.003	0.394	1.000
	0.197	1.003	0.786	0.199	1.005	0.198	1.004	0.197	1.003	0.196	1.001	0.196	1.000
0.25	0.797	1.069	0.098	0.840	1.106	0.815	1.091	0.740	1.067	0.765	1.033	0.754	1.000
	0.599	1.100	0.197	0.628	1.167	0.612	1.134	0.595	1.094	0.578	1.046	0.570	1.000
	0.392	1.090	0.336	0.409	1.160	0.400	1.126	0.389	1.087	0.378	1.042	0.373	1.000
	0.199	1.053	0.582	0.210	1.092	0.204	1.071	0.198	1.048	0.191	1.023	0.188	1.000
2.5	0.799	1.089	0.092	0.895	1.055	0.868	1.074	0.804	1.113	0.683	1.106	0.610	1.000
	0.596	1.171	0.177	0.768	1.128	0.690	1.184	0.574	1.214	0.460	1.147	0.409	1.000
	0.396	1.244	0.264	0.589	1.245	0.469	1.301	0.365	1.283	0.294	1.170	0.264	1.000
	0.198	1.306	0.376	0.305	1.423	0.226	1.405	0.182	1.327	0.156	1.180	0.142	1.000
25.0	0.797	1.098	0.089	0.901	1.050	0.880	1.060	0.838	1.083	0.670	1.183	0.365	1.000
	0.600	1.190	0.166	0.797	1.102	0.752	1.126	0.657	1.180	0.343	1.336	0.160	1.000
	0.398	1.282	0.252	0.657	1.173	0.579	1.215	0.422	1.308	0.132	1.410	0.067	1.000
	0.195	1.372	0.368	0.442	1.283	0.332	1.345	0.166	1.440	0.035	1.424	0.022	1.000

Table 24. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.800	1.013	0.106	0.844	1.021	0.818	1.018	0.792	1.013	0.766	1.006	0.754	1.000
	0.596	1.019	0.233	0.630	1.032	0.610	1.025	0.590	1.017	0.571	1.008	0.562	1.000
	0.394	1.016	0.410	0.417	1.029	0.404	1.021	0.390	1.015	0.378	1.007	0.372	1.000
	0.197	1.009	0.721	0.209	1.015	0.201	1.012	0.194	1.008	0.188	1.004	0.185	1.000
2.5	0.800	1.018	0.105	0.891	1.011	0.864	1.015	0.804	1.023	0.686	1.019	0.688	1.000
	0.596	1.034	0.225	0.752	1.028	0.679	1.039	0.574	1.041	0.458	1.025	0.400	1.000
	0.398	1.047	0.369	0.560	1.055	0.463	1.060	0.370	1.051	0.287	1.027	0.250	1.000
	0.199	1.057	0.584	0.287	1.088	0.231	1.076	0.185	1.055	0.142	1.027	0.124	1.000
25.0	0.799	1.019	0.105	0.893	1.011	0.872	1.013	0.830	1.017	0.674	1.036	0.380	1.000
	0.598	1.038	0.224	0.768	1.023	0.723	1.028	0.630	1.039	0.361	1.061	0.164	1.000
	0.397	1.056	0.371	0.613	1.039	0.539	1.048	0.392	1.066	0.157	1.069	0.065	1.000
	0.194	1.073	0.595	0.392	1.062	0.294	1.074	0.152	1.009	0.047	1.066	0.018	1.000

Table 25. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.800	1.008	0.105	0.805	1.013	0.802	1.010	0.799	1.007	0.797	1.003	0.795	1.000
	0.594	1.006	0.246	0.597	1.009	0.595	1.007	0.593	1.005	0.591	1.002	0.590	1.000
	0.399	1.004	0.436	0.402	1.006	0.400	1.005	0.399	1.003	0.398	1.002	0.397	1.000
	0.201	1.002	0.770	0.202	1.003	0.201	1.002	0.201	1.002	0.200	1.001	0.199	1.000
0.25	0.792	1.043	0.094	0.834	1.065	0.810	1.056	0.785	1.042	0.762	1.021	0.750	1.00
	0.597	1.065	0.176	0.618	1.107	0.607	1.086	0.595	1.061	0.580	1.030	0.572	1.00
	0.395	1.067	0.281	0.403	1.112	0.401	1.089	0.395	1.063	0.387	1.031	0.382	1.00
	0.198	1.040	0.481	0.206	1.069	0.202	1.053	0.197	1.036	0.191	1.017	0.189	1.00
2.5	0.799	1.054	0.084	0.901	1.031	0.873	1.043	0.802	1.069	0.672	1.067	0.601	1.00
	0.599	1.104	0.148	0.795	1.067	0.707	1.105	0.562	1.134	0.444	1.096	0.401	1.00
	0.399	1.152	0.205	0.655	1.120	0.482	1.179	0.338	1.183	0.279	1.116	0.260	1.00
	0.197	1.198	0.265	0.403	1.220	0.219	1.253	0.155	1.218	0.151	1.128	0.149	1.00
25.0	0.800	1.050	0.076	0.911	1.027	0.892	1.033	0.851	1.046	0.648	1.118	0.302	1.00
	0.600	1.114	0.136	0.828	1.052	0.785	1.065	0.683	1.100	0.210	1.237	0.094	1.00
	0.393	1.172	0.198	0.712	1.087	0.628	1.114	0.405	1.192	0.030	1.271	0.025	1.00
	0.194	1.227	0.272	0.527	1.144	0.372	1.194	0.090	1.287	0.003	1.268	0.006	1.00

Table 26. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.5$ $E_a = 14$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0 W	r = 0 T	r = 0.2 W	r = 0.2 T	r = 0.3 W	r = 0.3 T	r = 0.4 W	r = 0.4 T	r = 0.5 W	r = 0.5 T
0.025	0.797	1.013	0.102	0.802	1.022	0.799	1.017	0.796	1.012	0.794	1.006	0.792	1.000
	0.600	1.010	0.229	0.604	1.016	0.602	1.013	0.600	1.009	0.598	1.004	0.597	1.000
	0.399	1.006	0.419	0.401	1.011	0.400	1.008	0.398	1.006	0.397	1.003	0.396	1.000
	0.201	1.003	0.747	0.202	1.005	0.201	1.004	0.201	1.003	0.200	1.001	0.200	1.000
0.10	0.801	1.059	0.082	0.833	1.097	0.816	1.078	0.796	1.054	0.776	1.026	0.765	1.000
	0.601	1.069	0.156	0.613	1.115	0.609	1.092	0.600	1.065	0.558	1.031	0.581	1.000
	0.398	1.047	0.270	0.411	1.079	0.405	1.062	0.397	1.043	0.397	1.020	0.381	1.000
	0.197	1.019	0.532	0.208	1.034	0.202	1.026	0.195	1.017	0.189	1.008	0.186	1.000
0.15	0.795	1.061	0.084	0.831	1.097	0.811	1.080	0.790	1.058	0.769	1.029	0.758	1.000
	0.599	1.089	0.150	0.604	1.146	0.604	1.118	0.599	1.084	0.589	1.042	0.582	1.000
	0.399	1.084	0.231	0.395	1.138	0.401	1.111	0.401	1.079	0.396	1.039	0.392	1.000
	0.197	1.034	0.428	0.205	1.059	0.201	1.045	0.196	1.031	0.190	1.015	0.187	1.000
0.25	0.796	1.073	0.080	0.836	1.107	0.812	1.095	0.789	1.073	0.768	1.038	0.759	1.000
	0.601	1.125	0.131	0.596	1.200	0.602	1.164	0.604	1.121	0.599	1.063	0.594	1.000
	0.399	1.161	0.178	0.353	1.260	0.386	1.217	0.410	1.161	0.421	1.091	0.420	1.000
	0.201	1.154	0.246	0.161	1.243	0.189	1.208	0.210	1.155	0.220	1.087	0.220	1.000
2.5	0.801	1.089	0.068	0.914	1.045	0.888	1.062	0.808	1.112	0.653	1.128	0.592	1.000
	0.599	1.177	0.103	0.843	1.083	0.762	1.141	0.540	1.244	0.387	1.211	0.388	1.000
	0.399	1.263	0.127	0.774	1.122	0.583	1.252	0.257	1.364	0.200	1.272	0.250	1.000
	0.197	1.348	0.150	0.668	1.186	0.258	1.420	0.006	1.442	0.008	1.306	0.139	1.000

Table 27. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		4 = 0.4		r = 0.5	
	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.798	1.014	0.102	0.842	1.021	0.816	1.018	0.790	1.013	0.765	1.006	0.754	1.00
	0.595	1.019	0.217	0.627	1.033	0.609	1.026	0.590	1.018	0.572	1.008	0.563	1.00
	0.399	1.018	0.369	0.420	1.030	0.408	1.023	0.396	1.016	0.383	1.008	0.378	1.00
	0.195	1.009	0.664	0.207	1.016	0.200	1.012	0.193	1.008	0.187	1.004	0.184	1.00
2.5	0.800	1.018	0.097	0.895	1.011	0.868	1.015	0.801	1.023	0.678	1.020	0.601	1.000
	0.597	1.034	0.195	0.767	1.026	0.684	1.038	0.566	1.042	0.450	1.026	0.396	1.000
	0.398	1.049	0.297	0.586	1.051	0.459	1.061	0.358	1.054	0.283	1.030	0.250	1.000
	0.199	1.062	0.425	0.297	1.088	0.233	1.080	0.182	1.061	0.150	1.031	0.134	1.000
25.0	0.797	1.019	0.096	0.899	1.010	0.878	1.012	0.835	1.017	0.647	1.039	0.332	1.000
	0.598	1.038	0.187	0.792	1.020	0.746	1.026	0.641	1.038	0.285	1.067	0.124	1.000
	0.398	1.056	0.289	0.658	1.035	0.575	1.044	0.377	1.068	0.090	1.074	0.040	1.000
	0.198	1.074	0.427	0.459	1.055	0.325	1.071	0.107	1.094	0.019	1.072	0.009	1.000

Table 28. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.799	1.008	0.102	0.803	1.013	0.801	1.010	0.796	1.007	0.795	1.003	0.794	1.00
	0.594	1.006	0.236	0.598	1.010	0.596	1.008	0.594	1.005	0.592	1.002	0.591	1.00
	0.396	1.004	0.426	0.398	1.006	0.397	1.005	0.396	1.003	0.394	1.002	0.394	1.00
	0.197	1.002	0.760	0.198	1.003	0.198	1.002	0.198	1.002	0.196	1.001	0.196	1.00
0.10	0.793	1.031	0.088	0.807	1.051	0.799	1.041	0.790	1.029	0.781	1.014	0.777	1.000
	0.595	1.034	0.174	0.603	1.056	0.599	1.044	0.594	1.031	0.588	1.015	0.585	1.000
	0.394	1.022	0.308	0.401	1.037	0.397	1.029	0.393	1.020	0.388	1.009	0.386	1.000
	0.199	1.009	0.583	0.204	1.015	0.201	1.012	0.198	1.008	0.196	1.004	0.194	1.000
0.25	0.795	1.044	0.082	0.835	1.065	0.811	1.057	0.788	1.043	0.767	1.022	0.756	1.000
	0.594	1.075	0.137	0.589	1.121	0.596	1.096	0.597	1.072	0.592	1.037	0.586	1.000
	0.395	1.096	0.186	0.351	1.150	0.375	1.110	0.412	1.072	0.414	1.048	0.413	1.000
	0.201	1.090	0.256	0.165	1.142	0.191	1.117	0.210	1.086	0.218	1.045	0.218	1.000
2.5	0.798	1.055	0.070	0.912	1.027	0.885	1.038	0.803	1.067	0.649	1.076	0.587	1.000
	0.601	1.106	0.106	0.843	1.050	0.760	1.086	0.535	1.147	0.391	1.122	0.385	1.000
	0.399	1.159	0.130	0.778	1.072	0.579	1.153	0.243	1.220	0.202	1.259	0.245	1.000
	0.198	1.210	0.152	0.689	1.105	0.242	1.258	0.046	1.265	0.080	1.179	0.136	1.000

Table 29. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.1 \quad E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.800	0.987	0.123	0.846	0.979	0.819	0.983	0.792	0.988	0.765	0.994	0.753	1.000
	0.593	0.984	0.299	0.631	0.972	0.608	0.979	0.586	0.986	0.565	0.993	0.556	1.000
	0.400	0.987	0.528	0.426	0.977	0.411	0.983	0.396	0.989	0.382	0.995	0.376	1.000
	0.197	0.993	0.922	0.209	0.988	0.202	0.991	0.194	0.994	0.187	0.997	0.184	1.000
2.50	0.798	0.982	0.133	0.881	0.987	0.857	0.983	0.802	0.977	0.688	0.983	0.607	1.000
	0.596	0.967	0.348	0.735	0.967	0.674	0.960	0.576	0.963	0.454	0.981	0.393	1.000
	0.394	0.957	0.707	0.551	0.936	0.460	0.943	0.362	0.959	0.270	0.981	0.231	1.000
	0.193	0.955	1.384	0.296	0.919	0.229	0.941	0.170	0.962	0.122	0.983	0.104	1.000
25.00	0.796	0.980	0.144	0.876	0.988	0.855	0.985	0.817	0.981	0.692	0.967	0.405	1.000
	0.600	0.963	0.387	0.727	0.972	0.690	0.968	0.622	0.959	0.416	0.951	0.186	1.000
	0.396	0.946	0.873	0.542	0.953	0.494	0.946	0.401	0.934	0.194	0.954	0.072	1.000
	0.199	0.933	1.974	0.330	0.929	0.276	0.919	0.174	0.925	0.059	0.962	0.019	1.000

Table 30. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.3$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.798	0.962	0.143	0.844	0.937	0.817	0.950	0.789	0.966	0.762	0.985	0.750	1.000
	0.596	0.957	0.360	0.633	0.924	0.610	0.943	0.588	0.962	0.568	0.983	0.558	1.000
	0.395	0.967	0.655	0.420	0.942	0.405	0.956	0.390	0.971	0.376	0.987	0.370	1.000
	0.200	0.981	1.090	0.213	0.966	0.205	0.975	0.197	0.983	0.190	0.992	0.187	1.000
2.50	0.795	0.946	0.171	0.874	0.959	0.853	0.946	0.798	0.931	0.683	0.955	0.601	1.000
	0.598	0.905	0.520	0.745	0.891	0.675	0.878	0.572	0.901	0.449	0.955	0.388	1.000
	0.400	0.880	1.155	0.553	0.796	0.460	0.842	0.365	0.897	0.275	0.957	0.235	1.000
	0.197	0.889	2.322	0.282	0.798	0.229	0.854	0.178	0.908	0.132	0.961	0.113	1.000
25.00	0.798	0.942	0.192	0.866	0.960	0.848	0.954	0.815	0.942	0.705	0.905	0.411	1.000
	0.601	0.889	0.716	0.708	0.910	0.681	0.900	0.629	0.867	0.402	0.884	0.174	1.000
	0.399	0.845	2.252	0.546	0.855	0.509	0.819	0.381	0.817	0.168	0.912	0.061	1.000
	0.198	0.828	6.656	0.378	0.740	0.266	0.773	0.146	0.850	0.047	0.939	0.015	1.000

Table 31. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.800	0.987	0.127	0.846	0.979	0.819	0.983	0.791	0.988	0.765	0.995	0.753	1.000
	0.597	0.984	0.311	0.635	0.973	0.612	0.979	0.590	0.986	0.569	0.994	0.560	1.000
	0.397	0.988	0.565	0.423	0.978	0.407	0.984	0.392	0.989	0.379	0.995	0.373	1.000
	0.196	0.993	0.975	0.209	0.988	0.201	0.991	0.194	0.994	0.187	0.997	0.184	1.000
2.50	0.796	0.982	0.143	0.878	0.987	0.855	0.983	0.800	0.977	0.686	0.983	0.606	1.000
	0.598	0.968	0.384	0.737	0.967	0.677	0.960	0.579	0.964	0.457	0.982	0.395	1.000
	0.395	0.958	0.814	0.555	0.935	0.462	0.944	0.363	0.961	0.271	0.983	0.232	1.000
	0.200	0.958	1.592	0.305	0.923	0.237	0.944	0.177	0.964	0.128	0.984	0.109	1.000
25.00	0.801	0.981	0.150	0.876	0.988	0.857	0.986	0.821	0.981	0.706	0.968	0.423	1.000
	0.597	0.963	0.457	0.717	0.971	0.682	0.967	0.621	0.958	0.419	0.954	0.189	1.000
	0.398	0.947	1.097	0.537	0.952	0.496	0.945	0.407	0.934	0.196	0.958	0.074	1.000
	0.199	0.936	1.274	0.355	0.927	0.279	0.919	0.170	0.931	0.057	0.967	0.018	1.000

Table 32. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.3$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.799	0.963	0.156	0.846	0.939	0.818	0.952	0.791	0.967	0.764	0.985	0.752	1.000
	0.600	0.960	0.401	0.636	0.930	0.615	0.948	0.593	0.966	0.572	0.985	0.563	1.000
	0.400	0.970	0.729	0.425	0.947	0.410	0.960	0.395	0.974	0.381	0.988	0.375	1.000
	0.198	0.982	1.221	0.211	0.969	0.203	0.977	0.196	0.985	0.189	0.993	0.186	1.000
2.50	0.799	0.948	0.197	0.875	0.959	0.856	0.946	0.803	0.933	0.689	0.958	0.608	1.000
	0.597	0.907	0.668	0.750	0.881	0.672	0.879	0.569	0.907	0.451	0.959	0.390	1.000
	0.399	0.885	1.487	0.536	0.799	0.454	0.849	0.369	0.903	0.284	0.960	0.244	1.000
	0.193	0.904	2.962	0.266	0.827	0.223	0.873	0.178	0.919	0.135	0.965	0.116	1.000
25.00	0.796	0.942	0.246	0.858	0.957	0.842	0.952	0.814	0.942	0.710	0.908	0.418	1.000
	0.598	0.890	1.116	0.707	0.909	0.687	0.897	0.628	0.861	0.389	0.898	0.170	1.000
	0.396	0.850	3.779	0.582	0.851	0.511	0.809	0.361	0.835	0.161	0.928	0.060	1.000
	0.198	0.837	10.025	0.372	0.717	0.263	0.786	0.151	0.867	0.052	0.947	0.017	1.000

Table 33. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.1$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.795	1.225	0.007	0.961	1.053	0.885	1.145	0.768	1.257	0.616	1.386	0.534	1.500
	0.595	1.304	0.013	0.814	1.158	0.687	1.244	0.548	1.332	0.412	1.423	0.351	1.500
	0.399	1.361	0.020	0.591	1.252	0.470	1.317	0.356	1.382	0.259	1.447	0.219	1.500
	0.199	1.414	0.030	0.313	1.344	0.238	1.386	0.174	1.427	0.123	1.467	0.104	1.500
0.25	0.798	1.093	0.018	0.982	0.998	0.957	1.007	0.823	1.076	0.457	1.273	0.191	1.500
	0.601	1.144	0.040	0.941	0.995	0.797	1.045	0.533	1.165	0.207	1.342	0.072	1.500
	0.396	1.200	0.073	0.786	1.008	0.547	1.111	0.288	1.242	0.087	1.383	0.027	1.500
	0.195	1.276	0.122	0.479	1.079	0.271	1.196	0.114	1.308	0.028	1.414	0.008	1.500
2.50	0.799	1.034	0.054	0.949	0.995	0.939	0.994	0.889	0.993	0.447	1.137	0.008	1.500
	0.600	1.061	0.143	0.876	0.987	0.828	0.980	0.593	1.027	0.121	1.249	0.000	1.500
	0.399	1.097	0.289	0.773	0.971	0.603	0.987	0.273	1.109	0.025	1.314	0.000	1.500
	0.199	1.150	0.545	0.567	0.947	0.290	1.048	0.068	1.199	0.003	1.361	0.000	1.500

Table 34. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.3$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.800	1.194	0.007	0.959	1.025	0.885	1.107	0.772	1.224	0.623	1.372	0.541	1.500
	0.598	1.253	0.014	0.804	1.078	0.685	1.179	0.551	1.290	0.417	1.407	0.357	1.500
	0.397	1.304	0.023	0.573	1.149	0.463	1.244	0.355	1.337	0.260	1.429	0.220	1.500
	0.197	1.366	0.035	0.302	1.255	0.234	1.323	0.173	1.388	0.123	1.450	0.104	1.500
0.25	0.795	1.059	0.020	0.980	0.993	0.950	0.981	0.811	1.024	0.458	1.244	0.190	1.500
	0.598	1.085	0.047	0.928	0.946	0.774	0.970	0.524	1.105	0.211	1.323	0.073	1.500
	0.394	1.122	0.089	0.739	0.875	0.524	1.015	0.280	1.190	0.090	1.369	0.028	1.500
	0.193	1.192	0.157	0.426	0.930	0.263	1.104	0.119	1.261	0.031	1.398	0.009	1.500
2.5	0.799	1.000	0.064	0.944	0.983	0.934	0.978	0.880	0.946	0.450	1.089	0.008	1.500
	0.600	1.003	0.189	0.867	0.956	0.813	0.912	0.572	0.952	0.119	1.240	0.009	1.500
	0.400	1.026	0.415	0.773	0.879	0.573	0.868	0.260	1.068	0.022	1.316	0.002	1.500
	0.198	1.081	0.835	0.519	0.770	0.273	0.970	0.064	1.184	0.002	1.362	0.000	1.500

Table 35. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.795	1.118	0.003	0.996	1.000	0.957	1.026	0.807	1.118	0.458	1.305	0.197	1.500
	0.596	1.180	0.006	0.945	1.012	0.783	1.088	0.527	1.210	0.215	1.366	0.077	1.500
	0.400	1.240	0.010	0.772	1.056	0.549	1.162	0.303	1.279	0.097	1.402	0.031	1.500
	0.197	1.309	0.016	0.466	1.151	0.277	1.250	0.123	1.343	0.032	1.430	0.009	1.500
0.25	0.800	1.061	0.010	0.990	0.999	0.981	0.998	0.874	1.021	0.378	1.214	0.007	1.500
	0.597	1.109	0.028	0.967	0.994	0.847	1.007	0.532	1.109	0.097	1.314	0.006	1.500
	0.398	1.159	0.053	0.864	0.980	0.593	1.056	0.244	1.197	0.021	1.363	0.000	1.500
	0.148	1.221	0.092	0.586	1.006	0.286	1.081	0.066	1.273	0.002	1.397	0.000	1.500
2.50	0.794	1.029	0.047	0.956	0.996	0.948	0.994	0.904	0.989	0.382	1.117	0.000	1.500
	0.601	1.054	0.126	0.893	0.989	0.855	0.981	0.607	1.010	0.050	1.239	0.000	1.500
	0.397	1.089	0.264	0.803	0.974	0.633	0.977	0.235	1.096	0.003	1.308	0.000	1.500
	0.199	1.141	0.505	0.625	0.943	0.293	1.034	0.032	1.192	0.000	1.356	0.000	1.500
25.00	0.795	1.000	0.113	0.903	0.990	0.887	0.989	0.857	0.985	0.653	0.984	0.000	1.500
	0.600	1.001	0.331	0.774	0.977	0.743	0.974	0.682	0.964	0.166	1.075	0.000	1.500
	0.395	1.012	0.792	0.611	0.960	0.570	0.954	0.401	0.958	0.005	1.183	0.000	1.500
	0.197	1.045	1.841	0.428	0.939	0.334	0.936	0.074	1.041	0.000	1.268	0.000	1.500

Table 36. Concentration and Temperature Profiles for Gases
Entrance Velocity Profile: Parabolic
$\Delta H = 0.3$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.025	0.796	1.083	0.003	0.995	0.996	0.949	0.995	0.800	1.065	0.469	1.275	0.206	1.500
	0.599	1.116	0.006	0.926	0.947	0.764	1.008	0.530	1.148	0.231	1.344	0.083	1.500
	0.399	1.159	0.011	0.715	0.918	0.526	1.061	0.311	1.221	0.109	1.383	0.035	1.500
	0.194	1.236	0.020	0.403	1.014	0.266	1.158	0.133	1.291	0.038	1.411	0.013	1.500
0.25	0.799	1.026	0.012	0.988	0.996	0.976	0.982	0.855	0.962	0.392	1.173	0.006	1.500
	0.599	1.046	0.034	0.957	0.962	0.815	0.927	0.525	1.040	0.110	1.300	0.015	1.500
	0.396	1.079	0.067	0.803	0.846	0.556	0.955	0.254	1.149	0.023	1.357	0.000	1.500
	0.198	1.140	0.123	0.491	0.835	0.281	1.041	0.083	1.233	0.005	1.387	0.001	1.500
2.50	0.797	0.994	0.058	0.952	0.985	0.943	0.981	0.894	0.942	0.407	1.065	0.000	1.500
	0.599	0.998	0.178	0.886	0.962	0.837	0.911	0.572	0.933	0.500	1.241	0.002	1.500
	0.398	1.020	0.398	0.808	0.886	0.589	0.873	0.226	1.064	0.006	1.315	0.000	1.500
	0.201	1.071	0.787	0.558	0.754	0.281	0.954	0.034	1.185	0.001	1.359	0.000	1.500
25.00	0.795	0.963	0.163	0.892	0.967	0.878	0.963	0.854	0.954	0.637	0.909	0.000	1.500
	0.598	0.942	0.643	0.767	0.929	0.747	0.920	0.674	0.869	0.098	1.095	0.000	1.500
	0.396	0.951	1.897	0.653	0.890	0.597	0.837	0.304	0.915	0.000	1.228	0.000	1.500
	0.194	1.003	4.816	0.529	0.780	0.283	0.860	0.017	1.098	0.000	1.311	0.000	1.500

Table 37. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1 \quad E_a = 7$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.798	1.010	0.118	0.884	1.013	0.862	1.012	0.823	1.010	0.689	1.006	0.404	1.00
	0.596	1.012	0.279	0.736	1.019	0.696	1.016	0.619	1.011	0.404	1.006	0.185	1.00
	0.398	1.010	0.514	0.561	1.016	0.504	1.013	0.401	1.010	0.200	1.004	0.078	1.00
	0.195	1.005	0.973	0.334	1.009	0.270	1.007	0.174	1.004	0.064	1.002	0.022	1.00
2.5	0.795	1.016	0.119	0.883	1.012	0.862	1.015	0.820	1.019	0.696	1.018	0.242	1.000
	0.594	1.029	0.270	0.742	1.028	0.696	1.033	0.615	1.034	0.422	1.023	0.048	1.000
	0.396	1.040	0.479	0.566	1.048	0.500	1.049	0.400	1.042	0.208	1.023	0.007	1.000
	0.195	1.045	0.840	0.328	1.070	0.266	1.059	0.185	1.043	0.062	1.021	0.000	1.000
25.0	0.797	1.020	0.115	0.887	1.011	0.866	1.013	0.826	1.017	0.701	1.030	0.125	1.000
	0.598	1.039	0.259	0.754	1.025	0.710	1.029	0.629	1.037	0.404	1.061	0.000	1.000
	0.398	1.058	0.457	0.588	1.041	0.522	1.048	0.409	1.059	0.163	1.085	0.000	1.000
	0.197	1.078	0.790	0.365	1.064	0.288	1.071	0.175	1.083	0.039	1.096	0.000	1.000

Table 38. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3 \quad E_a = 7$

				r = 0		r - 0.2		r - 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.796	1.030	0.111	0.880	1.040	0.858	1.037	0.818	1.031	0.691	1.018	0.412	1.000
	0.601	1.041	0.242	0.728	1.063	0.691	1.053	0.624	1.039	0.429	1.020	0.205	1.000
	0.399	1.036	0.439	0.538	1.059	0.493	1.047	0.409	1.033	0.221	1.016	0.091	1.000
	0.193	1.017	0.839	0.309	1.030	0.262	1.023	0.181	1.016	0.072	1.007	0.025	1.000
1.00	0.801	1.042	0.105	0.889	1.036	0.865	1.044	0.822	1.050	0.703	1.038	0.317	1.000
	0.600	1.076	0.223	0.745	1.088	0.694	1.092	0.615	1.082	0.442	1.049	0.115	1.000
	0.393	1.103	0.374	0.540	1.149	0.479	1.132	0.399	1.100	0.237	1.052	0.039	1.000
	0.194	1.110	0.603	0.281	1.185	0.247	1.145	0.198	1.100	0.093	1.048	0.000	1.000
2.50	0.801	1.048	0.102	0.892	1.033	0.870	1.041	0.825	1.056	0.695	1.058	0.269	1.000
	0.601	1.090	0.213	0.761	1.076	0.708	1.095	0.612	1.109	0.423	1.079	0.071	1.000
	0.399	1.130	0.344	0.590	1.135	0.502	1.156	0.386	1.146	0.223	1.085	0.017	1.000
	0.199	1.161	0.528	0.344	1.220	0.258	1.208	0.181	1.162	0.090	1.082	0.029	1.000

Table 39. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.5$ $E_a = 7$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
0.25	0.800	1.052	0.102	0.881	1.067	0.859	1.063	0.820	1.053	0.701	1.032	0.429	1.000
	0.596	1.077	0.216	0.711	1.120	0.677	1.099	0.618	1.073	0.443	1.038	0.221	1.000
	0.398	1.076	0.367	0.508	1.125	0.477	1.099	0.414	1.069	0.247	1.034	0.107	1.000
	0.197	1.039	0.675	0.283	1.066	0.256	1.051	0.197	1.035	0.090	1.016	0.034	1.000
1.00	0.801	1.072	0.095	0.892	1.057	0.867	1.073	0.819	1.086	0.700	1.068	0.333	1.000
	0.599	1.135	0.187	0.753	1.141	0.691	1.160	0.604	1.150	0.447	1.092	0.136	1.000
	0.395	1.191	0.285	0.550	1.256	0.471	1.243	0.390	1.193	0.262	1.103	0.058	1.000
	0.196	1.230	0.413	0.271	1.371	0.233	1.301	0.200	1.213	0.127	1.105	0.022	1.000
2.50	0.800	1.081	0.092	0.896	1.053	0.873	1.067	0.824	1.094	0.681	1.106	0.281	1.000
	0.596	1.158	0.176	0.776	1.117	0.716	1.155	0.597	1.196	0.399	1.152	0.089	1.000
	0.396	1.231	0.260	0.624	1.203	0.511	1.263	0.359	1.275	0.212	1.170	0.030	1.000
	0.200	1.296	0.362	0.396	1.337	0.258	1.375	0.158	1.317	0.094	1.170	0.009	1.000

Table 40. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.797	1.010	0.115	0.882	1.013	0.860	1.012	0.820	1.010	0.690	1.006	0.408	1.00
	0.599	1.013	0.259	0.733	1.020	0.694	1.017	0.622	1.012	0.418	1.006	0.196	1.00
	0.399	1.011	0.475	0.550	1.018	0.499	1.014	0.406	1.010	0.211	1.005	0.085	1.00
	0.200	1.005	0.888	0.329	1.009	0.273	1.007	0.183	1.005	0.071	1.002	0.024	1.00
2.5	0.797	1.016	0.111	0.887	1.012	0.864	1.014	0.821	1.019	0.694	0.019	0.254	1.000
	0.598	1.030	0.238	0.752	1.026	0.702	1.032	0.614	1.035	0.425	1.025	0.060	1.000
	0.400	1.042	0.397	0.597	1.047	0.502	1.051	0.396	1.046	0.222	1.026	0.012	1.000
	0.199	1.050	0.643	0.332	1.073	0.262	1.066	0.188	1.049	0.082	1.025	0.002	1.000
5.0	0.795	1.017	0.111	0.887	1.011	0.865	1.014	0.821	1.019	0.686	1.025	0.225	1.000
	0.597	1.033	0.233	0.758	1.025	0.709	1.031	0.615	1.039	0.407	1.034	0.038	1.000
	0.396	1.047	0.389	0.592	1.043	0.512	1.052	0.385	1.056	0.199	1.036	0.002	1.000
	0.200	1.059	0.614	0.368	1.068	0.272	1.074	0.172	1.064	0.070	1.035	0.000	1.000
10.0	0.800	1.018	0.106	0.892	1.011	0.870	1.013	0.829	1.018	0.692	1.029	0.213	1.000
	0.598	1.035	0.229	0.762	1.024	0.716	1.029	0.623	1.040	0.387	1.045	0.025	1.000
	0.392	1.051	0.385	0.598	1.041	0.521	1.050	0.381	1.063	0.168	1.049	0.000	1.000
	0.201	1.066	0.598	0.389	1.063	0.289	1.075	0.161	1.077	0.053	1.046	0.000	1.000
25.0	0.801	1.018	0.105	0.893	1.011	0.872	1.013	0.832	1.017	0.695	1.032	0.180	1.000
	0.595	1.037	0.228	0.764	1.024	0.718	1.028	0.628	1.038	0.362	1.059	0.011	1.000
	0.395	1.055	0.376	0.608	1.039	0.534	1.047	0.397	1.063	0.135	1.068	0.000	1.000
	0.196	1.071	0.596	0.394	1.061	0.298	1.072	0.154	1.086	0.030	1.065	0.000	1.000

Table 41. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	ΔT	W	ΔT	W	ΔT	W	T	W	T
0.25	0.798	1.032	0.098	0.880	1.041	0.856	0.039	0.817	1.033	0.702	1.020	0.434	1.00
	0.600	1.050	0.197	0.705	1.077	0.674	1.064	0.621	1.047	0.462	1.025	0.237	1.00
	0.394	1.054	0.324	0.476	1.090	0.459	1.070	0.416	1.049	0.267	1.024	0.121	1.00
	0.197	1.031	0.567	0.249	1.052	0.244	1.040	0.208	1.027	0.106	1.013	0.042	1.00
1.00	0.796	1.045	0.092	0.892	1.034	0.865	1.044	0.813	1.054	0.690	1.043	0.336	1.000
	0.592	1.085	0.169	0.757	1.082	0.685	1.099	0.587	1.097	0.443	1.060	0.149	1.000
	0.391	1.124	0.238	0.565	1.151	0.457	1.156	0.373	1.129	0.278	1.070	0.076	1.000
	0.196	1.158	0.310	0.270	1.241	0.213	1.206	0.196	1.150	0.161	1.075	0.039	1.000
2.50	0.800	1.049	0.086	0.900	1.030	0.877	1.039	0.825	1.057	0.671	1.068	0.293	1.000
	0.597	1.097	0.154	0.794	1.064	0.731	1.088	0.589	1.123	0.378	1.102	0.108	1.000
	0.399	1.143	0.211	0.670	1.105	0.539	1.152	0.328	1.182	0.197	1.117	0.047	1.000
	0.199	1.189	0.272	0.480	1.171	0.266	1.234	0.117	1.217	0.090	1.121	0.020	1.000
10.00	0.801	1.053	0.080	0.909	1.028	0.888	1.034	0.844	1.048	0.647	1.106	0.206	1.000
	0.597	1.108	0.139	0.820	1.054	0.773	1.069	0.654	1.111	0.207	1.198	0.053	1.000
	0.400	1.160	0.195	0.715	1.087	0.624	1.116	0.362	1.206	0.034	1.214	0.016	1.000
	0.197	1.214	0.264	0.535	1.142	0.353	1.204	0.050	1.287	0.004	1.198	0.004	1.000

Table 42. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	ΔT	W	ΔT	W	ΔT	W	ΔT	W	T
0.25	0.801	1.010	0.108	0.883	1.013	0.862	1.012	0.823	1.010	0.698	1.006	0.420	1.00
	0.595	1.014	0.246	0.722	1.021	0.685	1.018	0.618	1.013	0.423	1.007	0.202	1.00
	0.399	1.012	0.436	0.535	1.020	0.492	1.016	0.409	1.011	0.223	1.005	0.092	1.00
	0.194	1.006	0.829	0.309	1.010	0.263	1.008	0.183	1.005	0.074	1.004	0.026	1.00
1.00	0.800	1.014	0.105	0.889	1.012	0.865	1.015	0.821	1.017	0.702	1.013	0.317	1.000
	0.596	1.026	0.223	0.742	1.029	0.689	1.031	0.610	1.028	0.440	1.016	0.115	1.000
	0.395	1.035	0.364	0.539	1.050	0.477	1.044	0.401	1.034	0.243	1.018	0.042	1.000
	0.195	1.038	0.573	0.271	1.064	0.244	1.050	0.203	1.035	0.101	1.017	0.012	1.000
2.50	0.800	1.016	0.102	0.891	1.011	0.869	1.014	0.824	1.019	0.693	1.020	0.269	1.000
	0.595	1.031	0.213	0.759	1.025	0.704	1.032	0.604	1.037	0.416	1.027	0.070	1.000
	0.396	1.044	0.336	0.591	1.045	0.498	1.053	0.378	1.050	0.224	1.029	0.018	1.000
	0.193	1.055	0.504	0.339	1.074	0.246	1.071	0.175	1.056	0.094	1.028	0.004	1.000
10.00	0.793	1.018	0.102	0.892	1.011	0.870	1.013	0.825	1.018	0.670	1.031	0.201	1.000
	0.597	1.035	0.201	0.776	1.023	0.728	1.028	0.626	1.040	0.353	1.049	0.034	1.000
	0.400	1.052	0.311	0.633	1.037	0.550	1.047	0.380	1.064	0.143	1.055	0.002	1.000
	0.193	1.068	0.467	0.419	1.060	0.293	1.075	0.128	1.083	0.039	1.053	0.000	1.000
25.00	0.800	1.018	0.097	0.898	1.010	0.877	1.012	0.835	1.017	0.685	1.033	0.166	1.000
	0.601	1.037	0.195	0.784	1.022	0.738	1.026	0.642	1.037	0.324	1.064	0.022	1.000
	0.397	1.055	0.307	0.640	1.036	0.560	1.045	0.397	1.063	0.088	1.075	0.075	1.000
	0.201	1.072	0.456	0.438	1.057	0.321	1.069	0.137	1.089	0.014	1.072	0.000	1.000

Table 43. Concentration and Temperature Profiles for Newtonian Liquids
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3 \quad E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	ΔT	W	ΔT	W	ΔT	W	ΔT	W	T
0.10	0.800	1.022	0.094	0.872	0.032	0.853	0.028	0.819	0.021	0.701	0.012	0.521	1.00
	0.598	1.025	0.199	0.682	1.040	0.670	1.032	0.617	1.023	0.460	1.012	0.314	1.00
	0.396	1.016	0.362	0.497	1.027	0.471	1.021	0.403	1.015	0.259	1.007	0.167	1.00
	0.199	1.006	0.708	0.297	1.011	0.257	1.009	0.186	1.006	0.099	1.003	0.059	1.00
0.50	0.795	1.041	0.084	0.889	1.038	0.857	1.046	0.805	0.048	0.698	1.033	0.042	1.000
	0.596	1.079	0.138	0.737	1.092	0.661	1.098	0.590	0.084	0.490	1.048	0.239	1.000
	0.397	1.120	0.174	0.487	1.172	0.416	1.156	0.398	0.117	0.358	1.061	0.168	1.000
	0.200	1.162	0.203	0.116	1.272	0.164	1.213	0.235	0.147	0.263	1.072	0.126	1.000
2.50	0.799	1.050	0.072	0.911	1.027	0.888	1.035	0.831	0.056	0.630	1.081	0.313	1.000
	0.598	1.101	0.109	0.842	1.049	0.787	1.069	0.596	0.131	0.268	1.141	0.154	1.000
	0.400	1.152	0.133	0.779	1.069	0.668	1.112	0.216	1.233	0.078	1.170	0.091	1.000
	0.198	1.203	0.161	0.677	1.102	0.340	1.229	0.589	1.271	0.015	1.166	0.049	1.000
1.00	0.801	1.045	0.077	0.902	1.031	0.874	1.042	0.814	1.056	0.684	1.049	0.369	1.000
	0.594	1.091	0.125	0.797	1.067	0.706	1.098	0.556	1.115	0.432	1.077	0.199	1.000
	0.398	1.138	0.153	0.684	1.109	0.482	1.165	0.305	1.167	0.282	1.096	0.135	1.000
	0.200	1.187	0.175	0.516	1.172	0.183	1.244	0.117	1.204	0.181	1.110	0.098	1.000
10.00	0.799	1.054	0.062	0.925	1.023	0.907	1.028	0.866	1.041	0.602	1.127	0.177	1.000
	0.597	1.109	0.095	0.870	1.039	0.833	1.051	0.727	1.087	0.004	1.126	0.037	1.000
	0.396	1.164	0.131	0.788	1.064	0.705	1.090	0.229	1.254	0.000	1.242	0.008	1.000
	0.196	1.220	0.170	0.646	1.107	0.398	1.190	0.000	1.303	0.000	1.223	0.002	1.000

Table 44. Concentration and Temperature Profiles for Non-Newtonian Liquids: n = 0.2
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.798	1.010	0.165	0.832	1.015	0.832	1.013	0.827	1.011	0.778	1.006	0.571	1.000
	0.601	1.012	0.362	0.645	1.020	0.651	1.017	0.645	1.013	0.558	1.007	0.345	1.000
	0.398	1.010	0.653	0.442	1.017	0.451	1.014	0.443	1.010	0.340	1.005	0.184	1.000
	0.196	1.005	1.188	0.234	1.008	0.241	1.007	0.226	1.005	0.145	1.003	0.068	1.000
5.0	0.799	1.017	0.157	0.839	1.016	0.838	1.016	0.830	1.017	0.780	1.020	0.474	1.000
	0.600	1.033	0.324	0.664	1.034	0.661	1.034	0.644	1.037	0.560	1.032	0.207	1.000
	0.393	1.048	0.531	0.456	1.055	0.452	1.056	0.429	1.055	0.351	1.040	0.074	1.000
	0.199	1.059	0.806	0.235	1.077	0.231	1.077	0.220	1.069	0.180	1.042	0.018	1.000
10.0	0.797	1.018	0.157	0.839	1.016	0.838	1.016	0.830	1.017	0.778	1.022	0.456	1.000
	0.601	1.035	0.319	0.669	1.033	0.667	1.034	0.650	1.036	0.553	1.040	0.190	1.000
	0.401	1.051	0.511	0.475	1.053	0.472	1.053	0.446	1.057	0.340	1.051	0.064	1.000
	0.196	1.066	0.786	0.249	1.075	0.244	1.077	0.218	1.077	0.156	1.055	0.011	1.000
50.0	0.799	1.019	0.154	0.843	1.016	0.842	1.016	0.835	1.017	0.785	1.022	0.407	1.000
	0.600	1.038	0.313	0.675	1.033	0.673	1.033	0.658	1.034	0.556	1.046	0.127	1.000
	0.395	1.056	0.504	0.482	1.052	0.479	1.052	0.457	1.055	0.316	1.069	0.030	1.000
	0.196	1.074	0.762	0.265	1.073	0.262	1.074	0.237	1.077	0.118	1.084	0.000	1.000

Table 45. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.2$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 14$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.798	1.032	0.142	0.820	1.050	0.823	1.044	0.822	1.035	0.787	1.021	0.599	1.000
	0.600	1.048	0.275	0.590	1.084	0.616	1.069	0.636	1.051	0.595	1.028	0.402	1.000
	0.397	1.048	0.448	0.344	1.086	0.394	1.070	0.438	1.051	0.404	1.027	0.251	1.000
	0.196	1.023	0.802	0.152	1.040	0.194	1.033	0.229	1.024	0.196	1.013	0.108	1.000
2.5	0.794	1.051	0.130	0.837	1.049	0.835	1.050	0.824	1.055	0.766	1.057	0.513	1.000
	0.599	1.098	0.219	0.664	1.101	0.658	1.105	0.628	1.112	0.554	1.094	0.314	1.000
	0.397	1.147	0.292	0.456	1.164	0.440	1.172	0.402	1.173	0.374	1.121	0.206	1.000
	0.200	1.193	0.364	0.215	1.238	0.194	1.244	0.184	1.225	0.227	1.141	0.136	1.000
10.0	0.797	1.055	0.123	0.848	1.046	0.847	1.046	0.838	1.049	0.769	1.072	0.446	1.000
	0.600	1.108	0.204	0.696	1.091	0.695	1.092	0.671	1.101	0.496	1.146	0.229	1.000
	0.397	1.163	0.270	0.526	1.142	0.520	1.145	0.470	1.165	0.231	1.207	0.129	1.000
	0.199	1.217	0.334	0.316	1.205	0.306	1.210	0.229	1.240	0.074	1.235	0.074	1.000
50.0	0.799	1.058	0.117	0.857	1.043	0.856	1.043	0.848	1.046	0.788	1.065	0.226	1.000
	0.595	1.116	0.197	0.713	1.086	0.711	1.087	0.691	1.093	0.523	1.149	0.030	1.000
	0.398	1.172	0.263	0.548	1.136	0.544	1.137	0.507	1.149	0.205	1.247	0.012	1.000
	0.198	1.230	0.332	0.324	1.203	0.318	1.205	0.263	1.223	0.026	1.295	0.004	1.000

Table 46. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.2$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.800	1.010	0.157	0.831	1.015	0.832	1.013	0.827	1.011	0.782	1.006	0.581	1.00
	0.597	1.013	0.342	0.630	1.022	0.640	1.018	0.640	1.014	0.564	1.007	0.355	1.00
	0.398	1.011	0.601	0.423	1.019	0.440	1.015	0.443	1.011	0.354	1.006	0.197	1.00
	0.195	1.005	1.105	0.217	1.009	0.231	1.007	0.226	1.005	0.152	1.003	0.074	1.00
2.5	0.795	1.017	0.152	0.835	1.016	0.834	1.017	0.825	1.018	0.775	1.018	0.492	1.000
	0.600	1.031	0.295	0.658	1.034	0.654	1.035	0.636	1.035	0.568	1.027	0.253	1.000
	0.400	1.044	0.452	0.445	1.056	0.440	1.056	0.427	1.051	0.381	1.032	0.124	1.000
	0.201	1.055	0.659	0.198	1.080	0.202	1.075	0.217	1.061	0.215	1.035	0.050	1.000
10.0	0.797	1.018	0.147	0.841	1.016	0.840	1.016	0.832	1.017	0.776	1.023	0.456	1.000
	0.597	1.035	0.285	0.671	1.033	0.669	1.033	0.649	1.036	0.537	1.042	0.202	1.000
	0.396	1.052	0.428	0.479	1.052	0.475	1.053	0.442	1.058	0.318	1.055	0.085	1.000
	0.199	1.068	0.600	0.262	1.074	0.255	1.076	0.217	1.079	0.148	1.062	0.030	1.000
50.0	0.797	1.019	0.144	0.844	1.016	0.844	1.016	0.836	1.016	0.783	1.022	0.371	1.000
	0.592	1.038	0.280	0.678	1.032	0.676	1.032	0.659	1.034	0.538	1.048	0.104	1.000
	0.399	1.057	0.412	0.500	1.050	0.498	1.050	0.471	1.053	0.295	1.072	0.034	1.000
	0.195	1.075	0.584	0.279	1.072	0.276	1.073	0.243	1.076	0.091	1.089	0.009	1.000

Table 47. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.2$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 21$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
0.25	0.796	1.035	0.126	0.809	1.054	0.814	1.048	0.817	1.038	0.792	1.023	0.620	1.000
	0.592	1.063	0.216	0.513	1.112	0.572	1.090	0.624	1.066	0.626	1.035	0.456	1.000
	0.400	1.083	0.291	0.206	1.151	0.321	1.121	0.439	1.087	0.491	1.044	0.352	1.000
	0.200	1.073	0.415	0.033	1.126	0.098	1.105	0.215	1.078	0.310	1.041	0.228	1.000
2.5	0.800	1.051	0.108	0.847	1.046	0.845	1.047	0.832	1.052	0.763	1.061	0.544	1.000
	0.596	1.104	0.163	0.685	1.095	0.677	1.099	0.627	1.116	0.512	1.113	0.373	1.000
	0.398	1.157	0.194	0.521	1.144	0.495	1.156	0.383	1.188	0.312	1.156	0.296	1.000
	0.199	1.211	0.215	0.323	1.204	0.264	1.227	0.128	1.259	0.163	1.188	0.247	1.000
10.0	0.800	1.055	0.001	0.861	1.042	0.860	1.042	0.851	1.045	0.768	1.073	0.422	1.000
	0.595	1.111	0.147	0.743	1.077	0.740	1.078	0.714	1.087	0.397	1.185	0.261	1.000
	0.393	1.168	0.178	0.614	1.116	0.608	1.118	0.547	1.140	0.026	1.276	0.112	1.000
	0.199	1.223	0.206	0.411	1.177	0.397	1.182	0.240	1.237	0.000	1.275	0.056	1.000
50.0	0.800	1.057	0.092	0.877	1.037	0.877	1.037	0.869	1.039	0.808	1.059	0.000	1.000
	0.596	1.116	0.141	0.762	1.071	0.760	1.072	0.739	1.078	0.504	1.157	0.000	1.000
	0.398	1.173	0.180	0.605	1.119	0.600	1.120	0.549	1.136	0.009	1.303	0.000	1.000
	0.198	1.232	0.209	0.384	1.185	0.374	1.188	0.263	1.222	0.000	1.304	0.000	1.000

Table 48. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 14$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
1.0	0.796	1.014	0.134	0.863	1.014	0.853	1.016	0.825	1.016	0.736	1.012	0.310	1.000
	0.594	1.025	0.291	0.695	1.032	0.676	1.031	0.628	1.026	0.495	1.016	0.648	1.000
	0.396	1.032	0.492	0.490	1.049	0.473	1.042	0.427	1.032	0.294	1.017	0.000	1.000
	0.195	1.031	0.830	0.244	1.054	0.245	1.043	0.218	1.030	0.121	1.015	0.000	1.000
5.0	0.799	1.017	0.128	0.870	1.013	0.860	1.014	0.832	1.018	0.732	1.023	0.299	1.000
	0.601	1.032	0.270	0.720	1.028	0.699	1.031	0.638	1.037	0.473	1.033	0.006	1.000
	0.393	1.048	0.452	0.531	1.048	0.495	1.053	0.411	1.056	0.256	1.038	0.000	1.000
	0.199	1.060	0.698	0.307	1.072	0.266	1.075	0.200	1.066	0.109	1.037	0.000	1.000
10.0	0.798	1.018	0.128	0.870	1.013	0.860	1.014	0.832	1.017	0.728	1.027	0.282	1.000
	0.595	1.035	0.270	0.720	1.028	0.699	1.031	0.638	1.038	0.451	1.044	0.050	1.000
	0.398	1.051	0.437	0.546	1.046	0.514	1.050	0.422	1.060	0.234	1.050	0.000	1.000
	0.199	1.066	0.678	0.325	1.068	0.283	1.075	0.194	1.077	0.087	1.050	0.000	1.000

Table 49. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 14$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0 W	r = 0 T	r = 0.2 W	r = 0.2 T	r = 0.3 W	r = 0.3 T	r = 0.4 W	r = 0.4 T	r = 0.5 W	r = 0.5 T
0.25	0.797	1.032	0.116	9.855	1.044	0.846	1.041	0.822	0.034	0.737	1.020	0.491	1.000
	0.599	1.050	0.230	0.657	1.080	0.657	1.066	0.634	1.049	0.515	1.026	0.291	1.000
	0.399	1.053	0.371	0.423	1.089	0.446	1.071	0.438	1.051	0.324	1.026	0.163	1.000
	0.200	1.029	0.648	0.208	1.050	0.234	1.040	0.229	1.028	0.142	1.014	0.063	1.000
2.5	0.792	1.051	0.106	0.873	1.038	0.861	1.043	0.824	1.056	0.703	1.065	0.357	1.000
	0.598	1.097	0.181	0.744	1.078	0.712	1.092	0.617	1.117	0.448	1.099	0.167	1.000
	0.398	1.145	0.246	0.588	1.127	0.516	1.156	0.373	1.177	0.261	1.120	0.086	1.000
	0.200	1.191	0.311	0.374	1.196	0.258	1.233	0.156	1.220	0.137	1.130	0.045	1.000
10.0	0.796	1.055	0.098	0.884	1.035	0.874	1.038	0.843	1.048	0.696	1.093	0.273	1.000
	0.600	1.107	0.165	0.776	1.067	0.753	1.075	0.674	1.103	0.320	1.182	0.097	1.000
	0.399	1.161	0.226	0.644	1.107	0.600	1.122	0.434	1.182	0.080	1.220	0.038	1.000
	0.196	1.215	0.295	0.445	1.167	0.362	1.197	0.123	1.273	0.013	1.216	0.014	1.000
50.0	0.801	1.056	0.091	0.894	1.032	0.885	1.034	0.858	1.043	0.732	1.083	0.037	1.000
	0.598	1.114	0.160	0.785	1.064	0.765	1.071	0.696	1.092	0.297	1.221	0.007	1.000
	0.398	1.172	0.226	0.644	1.107	0.604	1.119	0.463	1.163	0.018	1.293	0.001	1.000
	0.201	1.229	0.297	0.438	1.169	0.367	1.191	0.158	1.256	0.000	1.290	0.000	1.000

Table 50. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 0.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 21$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0 W	r = 0 T	r = 0.2 W	r = 0.2 T	r = 0.3 W	r = 0.3 T	r = 0.4 W	r = 0.4 T	r = 0.5 W	r = 0.5 T
0.25	0.899	1.006	0.063	0.935	1.007	0.930	1.007	0.917	1.007	0.867	1.005	0.660	1.000
	0.799	1.010	0.129	0.861	1.014	0.852	1.013	0.827	1.010	0.732	1.006	0.473	1.000
	0.595	1.014	0.286	0.683	1.022	0.673	1.018	0.633	1.013	0.478	1.007	0.250	1.000
	0.397	1.012	0.506	0.485	1.020	0.477	1.016	0.430	1.011	0.269	1.006	0.122	1.000
	0.195	1.006	0.941	0.268	1.010	0.258	1.008	0.208	1.005	0.100	1.003	0.039	1.000
1.0	0.797	1.014	0.126	0.865	1.014	0.854	1.015	0.825	1.017	0.733	1.013	0.382	1.000
	0.596	1.026	0.260	0.699	1.032	0.676	1.032	0.627	1.028	0.496	1.017	0.163	1.000
	0.396	1.035	0.417	0.481	1.052	0.464	1.046	0.425	1.035	0.303	1.019	0.695	1.000
	0.196	1.038	0.653	0.219	1.064	0.232	1.052	0.223	1.037	0.142	1.018	0.023	1.000
5.0	0.794	1.017	0.123	0.869	1.013	0.859	1.014	0.829	1.018	0.717	1.024	0.311	1.000
	0.597	1.033	0.241	0.727	1.028	0.704	1.031	0.633	1.038	0.453	1.036	0.095	1.000
	0.401	1.048	0.369	0.557	1.045	0.514	1.051	0.410	1.058	0.251	1.042	0.025	1.000
	0.199	1.063	0.536	0.330	1.069	0.271	1.076	0.185	1.072	0.110	1.043	0.004	1.000
10.0	0.800	1.018	0.118	0.875	1.013	0.885	1.014	0.837	1.017	0.724	1.027	0.294	1.000
	0.597	1.035	0.236	0.733	1.027	0.712	1.029	0.644	1.037	0.426	1.047	0.007	1.000
	0.401	1.052	0.359	0.571	1.043	0.534	1.048	0.423	1.060	0.206	1.056	0.016	1.000
	0.193	1.069	0.526	0.347	1.066	0.292	1.074	0.172	1.082	0.070	1.057	0.000	1.000

Table 51. Concentration and Temperature Profiles for Non-Newtonian Liquids: n = 0.5
Entrance Velocity Profiles: Parabolic
$\Delta H = -0.3$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.796	1.035	0.004	0.651	1.047	0.840	1.044	0.818	1.037	0.743	1.022	0.511	1.000
	0.600	1.061	0.179	0.621	1.099	0.636	1.082	0.633	1.060	0.554	1.032	0.340	1.000
	0.400	1.085	0.245	0.312	1.148	0.396	1.115	0.450	1.081	0.408	1.040	0.240	1.000
	0.194	1.087	0.337	0.067	1.147	0.148	1.119	0.240	1.084	0.251	1.042	0.146	1.000
2.50	0.799	1.050	0.087	0.887	1.034	0.876	1.038	0.836	1.053	0.688	1.073	0.391	1.000
	0.595	1.102	0.132	0.788	1.064	0.755	1.078	0.620	1.122	0.356	1.133	0.216	1.000
	0.400	1.153	0.158	0.700	1.091	0.627	1.121	0.319	1.209	0.146	1.170	0.147	1.000
	0.196	1.207	0.184	0.570	1.132	0.378	1.206	0.031	1.276	0.040	1.182	0.098	1.000
10.00	0.794	1.056	0.079	0.900	1.030	0.891	1.033	0.861	1.043	0.670	1.106	0.239	1.000
	0.596	1.110	0.115	0.830	1.051	0.811	1.057	0.738	1.082	0.048	1.264	0.076	1.000
	0.397	1.165	0.150	0.730	1.081	0.688	1.095	0.464	1.177	0.000	1.257	0.022	1.000
	0.198	1.220	0.187	0.558	1.133	0.448	1.170	0.006	1.306	0.000	1.243	0.753	1.000

Table 52. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 1.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 14$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
0.25	0.792	1.019	0.107	0.889	1.013	0.859	1.012	0.812	1.010	0.665	1.006	0.378	1.00
	0.596	1.013	0.241	0.748	1.020	0.695	1.016	0.610	1.012	0.391	1.006	0.176	1.00
	0.395	1.011	0.446	0.568	1.018	0.496	1.014	0.388	1.010	0.188	1.005	0.072	1.00
	0.200	1.006	0.826	0.350	1.009	0.273	1.007	0.172	1.005	0.062	1.002	0.021	1.00
5.0	0.800	1.017	0.098	0.900	1.010	0.871	1.013	0.823	1.019	0.677	1.025	0.206	1.000
	0.598	1.033	0.215	0.777	1.023	0.713	1.031	0.605	1.040	0.381	1.034	0.027	1.000
	0.400	1.047	0.357	0.623	1.040	0.518	1.052	0.372	1.056	0.178	1.036	0.000	1.000
	0.197	1.059	0.578	0.395	1.067	0.266	1.074	0.155	1.063	0.056	1.033	0.000	1.000
10.0	0.798	1.018	0.098	0.900	1.010	0.872	1.013	0.823	1.018	0.670	1.031	0.181	1.000
	0.597	1.035	0.212	0.781	1.022	0.719	1.029	0.611	1.041	0.358	1.046	0.015	1.000
	0.395	1.051	0.354	0.628	1.038	0.526	1.050	0.366	1.064	0.147	1.049	0.000	1.000
	0.198	1.066	0.563	0.417	1.061	0.283	1.076	0.141	1.077	0.040	1.045	0.000	1.000
50.0	0.797	1.019	0.097	0.901	1.010	0.874	1.013	0.826	1.017	0.673	1.034	0.111	1.000
	0.601	1.037	0.205	0.788	1.021	0.729	1.027	0.628	1.038	0.334	1.067	0.000	1.000
	0.393	1.057	0.350	0.635	1.037	0.536	1.047	0.379	1.064	0.092	1.081	0.000	1.000
	0.199	1.074	0.555	0.430	1.058	0.301	1.071	0.141	1.088	0.013	1.078	0.000	1.000

Table 53. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 1.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 14$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
				W	T	W	T	W	T	W	T	W	T
0.25	0.797	1.032	0.091	0.889	1.040	0.856	1.039	0.811	1.033	0.684	1.020	0.410	1.000
	0.600	1.050	0.181	0.727	1.076	0.678	1.063	0.613	1.047	0.438	1.024	0.217	1.000
	0.397	1.054	0.297	0.507	1.090	0.467	1.070	0.408	1.048	0.248	1.024	0.108	1.000
	0.196	1.031	0.527	0.272	1.053	0.246	1.040	0.197	1.027	0.095	1.013	0.036	1.000
2.5	0.794	1.050	0.080	0.907	1.029	0.876	1.039	0.814	1.060	0.639	1.071	0.252	1.000
	0.599	1.096	0.140	0.816	1.058	0.738	1.087	0.578	1.126	0.350	1.102	0.089	1.000
	0.397	1.143	0.195	0.702	1.097	0.539	1.154	0.301	0.185	0.171	1.116	0.035	1.000
	0.196	1.189	0.255	0.524	1.162	0.247	1.237	0.098	1.215	0.073	1.117	0.013	1.000
10.0	0.799	1.054	0.073	0.917	1.025	0.891	1.033	0.840	1.050	0.615	1.114	0.172	1.000
	0.599	1.107	0.127	0.840	1.049	0.780	1.067	0.644	1.115	0.170	1.202	0.041	1.000
	0.399	1.160	0.180	0.743	1.079	0.626	1.116	0.315	1.221	0.023	1.210	0.010	1.000
	0.240	1.201	0.229	0.628	1.114	0.428	1.181	0.064	1.283	0.004	1.196	0.003	1.000
50.0	0.798	1.057	0.069	0.923	1.023	0.898	1.031	0.852	1.045	0.649	1.111	0.010	1.000
	0.598	1.114	0.125	0.844	1.047	0.786	1.065	0.663	1.102	0.076	1.280	0.001	1.000
	0.396	1.172	0.184	0.736	1.079	0.619	1.115	0.345	1.200	0.000	1.287	0.000	1.000
	0.197	1.229	0.255	0.563	1.132	0.336	1.201	0.357	1.293	0.000	1.275	0.000	1.000

Table 54. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 1.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.1$ $E_a = 21$

				r = 0		r = 0.2		r = 0.3		r = 0.4		r = 0.5	
$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	W	T	W	T	W	T	W	T	W	T
0.25	0.794	1.010	0.102	0.889	1.013	0.859	1.012	0.813	1.010	0.671	1.006	0.387	1.000
	0.595	1.018	0.226	0.741	1.021	0.688	1.018	0.609	1.013	0.399	1.007	0.184	1.000
	0.396	1.012	0.408	0.555	1.020	0.490	1.016	0.393	1.011	0.200	1.005	0.079	1.000
	0.196	1.006	0.770	0.331	1.010	0.263	1.008	0.173	1.005	0.064	1.002	0.022	1.000
2.5	0.796	1.016	0.095	0.899	1.010	0.869	1.014	0.816	1.020	0.670	1.020	0.234	1.000
	0.600	1.030	0.194	0.782	1.023	0.712	1.032	0.598	1.037	0.396	1.027	0.058	1.000
	0.398	1.044	0.310	0.623	1.043	0.502	1.052	0.364	1.049	0.201	1.028	0.013	1.000
	0.196	1.055	0.468	0.377	1.072	0.248	1.070	0.164	1.054	0.079	1.027	0.002	1.000
10.0	0.796	1.018	0.092	0.903	1.010	0.874	1.013	0.824	1.018	0.653	1.033	0.177	1.000
	0.596	1.035	0.185	0.795	1.021	0.732	1.028	0.613	1.041	0.320	1.050	0.002	1.000
	0.398	1.052	0.289	0.661	1.035	0.551	1.047	0.355	1.067	0.120	1.054	0.000	1.000
	0.195	1.068	0.435	0.458	1.057	0.291	1.075	0.110	1.084	0.030	1.051	0.000	1.000
50.0	0.796	1.019	0.089	0.906	1.009	0.879	1.012	0.831	1.017	0.664	1.035	0.078	1.000
	0.597	1.038	0.179	0.803	1.020	0.742	1.026	0.634	1.037	0.272	1.074	0.003	1.000
	0.400	1.056	0.283	0.671	1.033	0.567	1.044	0.384	1.063	0.048	1.087	0.000	1.000
	0.194	1.075	0.436	0.464	1.054	0.307	1.070	0.109	1.091	0.003	1.083	0.000	1.000

Table 55. Concentration and Temperature Profiles for Non-Newtonian Liquids: $n = 1.5$
Entrance Velocity Profile: Parabolic
$\Delta H = -0.3$ $E_a = 21$

$\frac{\alpha Pr}{Sc}$	W_M	T_M	Z^*	r = 0 W	r = 0 T	r = 0.2 W	r = 0.2 T	r = 0.3 W	r = 0.3 T	r = 0.4 W	r = 0.4 T	r = 0.5 W	r = 0.5 T
0.25	0.795	1.034	0.081	0.888	1.041	0.853	1.041	0.805	1.036	0.689	1.022	0.426	1.000
	0.599	1.061	0.143	0.707	1.091	0.659	1.077	0.609	1.057	0.475	1.030	0.256	1.000
	0.399	1.084	0.198	0.428	1.141	0.433	1.108	0.427	1.075	0.326	1.036	0.165	1.000
	0.197	1.093	0.264	0.132	1.159	0.194	1.121	0.241	1.082	0.196	1.039	0.097	1.000
2.5	0.795	1.051	0.066	0.920	1.025	0.891	1.034	0.824	1.059	0.597	1.086	0.280	1.000
	0.598	1.101	0.098	0.863	1.043	0.796	1.067	0.578	1.138	0.230	1.145	0.132	1.000
	0.401	1.151	0.121	0.809	1.060	0.679	1.111	0.165	1.242	0.060	1.167	0.074	1.000
	0.194	1.203	0.150	0.712	1.093	0.263	1.235	0.002	1.266	0.009	1.157	0.034	1.000
10.0	0.794	1.055	0.057	0.933	1.020	0.910	1.027	0.864	1.042	0.544	1.144	0.147	1.000
	0.597	1.109	0.087	0.885	1.035	0.837	1.050	0.713	1.093	0.001	1.256	0.026	1.000
	0.399	1.163	0.121	0.813	1.057	0.709	1.090	0.107	1.286	0.000	1.236	0.005	1.000
	0.199	1.219	0.160	0.687	1.096	0.363	1.204	0.000	1.301	0.000	1.216	0.000	1.000
50.0	0.798	1.057	0.052	0.939	1.018	0.919	1.024	0.879	1.036	0.654	1.112	0.000	1.000
	0.593	1.117	0.091	0.878	1.037	0.826	1.052	0.698	1.092	0.000	1.298	0.000	1.000
	0.397	1.174	0.127	0.800	1.060	0.686	1.095	0.217	1.240	0.000	1.294	0.000	1.000
	0.193	1.233	0.166	0.667	1.100	0.341	1.200	0.000	1.302	0.000	1.289	0.000	1.000

APPENDIX G

HEAT AND MASS TRANSFER RESULTS

Heat transfer results for non-Newtonian fluids for constant properties and for variable rheological properties appear in this section. Developing flow results for constant properties also appear here.

Table 56. Heat Transfer Results for Non-Newtonian Fluids
Constant Rheological Properties

$E_v = 0$

ξ	T_m	Nu_L	Nu_{LN}	ξ	T_m	Nu_L	Nu_{LN}
	$n = 1.0$				$n = 1.5$		
0.00142	0.0192	18.2	27.2	0.00148	0.0204	18.2	25.3
0.00490	0.0428	11.9	17.9	0.00446	0.0417	12.1	17.4
0.0196	0.1043	7.38	11.2	0.0179	0.1016	7.46	10.9
0.0589	0.2060	4.93	7.06	0.0551	0.2044	5.22	7.55
0.1868	0.4039	4.07	5.54	0.1743	0.4013	3.94	5.35
0.4751	0.6536	3.68	4.46	0.4424	0.6501	3.56	4.32
0.9470	0.8518	3.65	4.03	0.8952	0.8530	3.53	3.89
1.471	0.9411	3.65	3.85	1.539	0.9555	3.53	3.68
	$n = 0.5$				$n = 0.2$		
0.0071	0.0206	21.4	29.2	0.00183	0.0210	28.5	34.8
0.00491	0.0408	14.3	20.4	0.00490	0.0406	18.3	25.4
0.0197	0.0991	8.75	12.7	0.0209	0.1020	10.6	15.5
0.0609	0.1999	6.04	8.78	0.0627	0.2005	7.29	10.7
0.2064	0.4066	4.43	6.07	0.2101	0.4008	5.22	7.31
0.5289	0.6570	3.98	4.86	0.5542	0.6513	4.56	5.70
1.0479	0.8520	3.94	4.38	1.134	0.8520	4.49	5.05
1.771	0.9528	3.94	4.14	1.921	0.9520	4.48	4.74

Table 57. Heat Transfer Results for Non-Newtonian Fluids Variable Rheological Properties

$E_v = 1$

ξ	T_m	Nu_L	Nu_{LN}	ξ	T_m	Nu_L	Nu_{LN}
	n = 1.0				n = 1.5		
0.00063	0.0149	31.8	47.6	0.00081	0.0201	30.1	45.1
0.00328	0.0401	16.5	24.8	0.00319	0.0411	15.9	23.9
0.0149	0.1009	9.45	14.1	0.0144	0.0999	8.44	11.9
0.0502	0.2065	6.15	9.23	0.0465	0.2035	5.97	8.9
0.1561	0.4035	4.52	6.61	0.1449	0.4001	4.44	6.41
0.4220	0.6400	3.71	4.84	0.4800	0.6602	3.78	4.8
0.8009	0.8320	3.51	4.46				
	n = 0.5				n = 0.2		
0.00076	0.0154	32.6	48.9	0.00094	0.0179	34.8	54.1
0.00385	0.0402	17.0	25.5	0.00369	0.0400	20.9	33.1
0.0167	0.0991	9.94	15.0	0.0171	0.1005	11.9	18.5
0.0525	0.2035	7.35	10.4	0.0548	0.2025	7.78	12.4
0.1738	0.4025	5.27	7.11	0.1745	0.4016	5.85	8.05
0.3765	0.6010	4.50	5.85	0.4590	0.6419	4.92	6.05

Table 58. Heat Transfer Results for Non-Newtonian Fluids Variable Rheological Properties

$E_v = 2$

ξ	T_m	Nu_L	Nu_{LN}	ξ	T_m	Nu_L	Nu_{LN}
		$n = 1.0$				$n = 0.5$	
0.00107	0.0240	30.3	45.3	0.00127	0.0251	31.8	47.7
0.00255	0.0398	21.1	31.7	0.00301	0.0402	21.7	32.5
0.0124	0.0990	11.2	16.8	0.0142	0.1010	12.1	18.0
0.0425	0.2001	6.99	10.5	0.0459	0.2017	7.89	11.8
0.1531	0.4018	5.14	6.71	0.1561	0.4007	5.31	7.86
0.4151	0.6441	3.85	4.98	0.4241	0.6439	3.95	5.85
		$n = 0.2$					
0.00095	0.0205	43.6	65.3				
0.00421	0.0499	24.3	36.4				
0.0143	0.1013	15.0	22.4				
0.0465	0.2019	9.79	14.6				
0.1557	0.3997	6.59	9.86				
0.5411	0.7287	4.83	7.23				

Table 59. Heat Transfer Results for Non-Newtonian Fluids Variable Rheological Properties

$E_v = 3$

ξ	T_m	Nu_L	Nu_{LN}	ξ	T_m	Nu_L	Nu_{LN}
		n = 1.0				n = 0.5	
0.00102	0.0270	35.6	53.5	0.00104	0.0249	38.7	58.1
0.00201	0.0403	27.3	40.5	0.00232	0.0392	27.8	41.2
0.00951	0.1001	15.3	22.2	0.0117	0.0996	14.7	21.6
0.0347	0.2006	8.76	12.9	0.0392	0.1999	9.31	13.7
0.1312	0.4016	5.40	7.84	0.1441	0.3979	5.74	8.43
0.3628	0.6421	3.90	5.66	0.4087	0.6426	4.12	6.05
		n = 0.2					
0.00109	0.0251	46.4	69.8				
0.00245	0.0401	33.7	49.9				
0.0125	0.1019	19.5	28.7				
0.0402	0.1999	11.4	16.7				
0.1482	0.4013	7.06	10.4				
0.4651	0.6499	4.60	6.77				

Table 60. Heat Transfer Results for Non-Newtonian Fluids
Constant Rheological Properties
Entrance Velocity Profile: Uniform

Pr = 100				Pr = 10			
ξ	T_m	Nu_L	Nu_{LN}	ξ	T_m	Nu_L	Nu_{LN}
			n = 1.5				
0.00137	0.0201	16.0	27.0	0.00151	0.0283	12.9	30.9
0.00391	0.0389	10.9	18.4	0.00482	0.0499	9.38	19.3
0.0101	0.0710	8.44	13.3	0.0117	0.0831	7.05	13.5
0.0337	0.1449	5.68	8.44	0.0471	0.1861	5.64	7.94
0.1061	0.2997	4.27	6.02	0.1015	0.2922	4.37	6.18
0.3067	0.5509	3.84	4.73	0.3051	0.5502	3.88	4.76
			n = 1.0				
0.00091	0.0200	20.0	44.5	0.00101	0.0251	20.9	50.2
0.00541	0.0499	11.2	18.9	0.00356	0.0499	14.0	28.8
0.0161	0.0921	7.61	12.0	0.01326	0.0971	8.04	15.4
0.121	0.3113	4.36	6.16	0.1139	0.2986	4.41	6.23
0.461	0.6448	3.65	4.49	0.3465	0.5507	3.76	4.61
			n = 0.5				
0.00172	0.0237	14.3	33.5	0.00144	0.0309	20.9	52.4
0.00417	0.0399	11.1	23.5	0.00398	0.0501	14.8	31.0
0.0121	0.0741	7.79	15.3	0.0107	0.0808	10.0	18.9
0.0399	0.1511	6.59	9.85	0.0351	0.1521	6.45	11.3
0.1077	0.2753	5.34	7.18	0.1215	0.2899	4.82	6.77
0.4201	0.5991	4.23	5.21	0.4200	0.5992	4.23	5.24
			n = 0.2				
0.00137	0.0253	19.9	56.4	0.00149	0.0451	37.5	92.8
0.00571	0.0503	12.9	26.9	0.00401	0.0659	24.1	51.0
0.0166	0.0907	8.96	17.2	0.0135	0.1051	12.8	24.6
0.0631	0.2009	7.52	10.6	0.0583	0.2029	8.30	11.7
0.4211	0.6004	5.32	6.51	0.2117	0.4061	6.05	7.40

APPENDIX H

SAMPLE CALCULATIONS

The dimensionless variables used in the main body of this work are useful for theoretical developments and correlation purposes. However, in practical situations, the data is available in terms of dimensional properties. Several calculations are presented here to illustrate the calculation of the dimensionless groups and the use of the correlations.

1. Consider a gas reaction occurring in a tubular reactor with the following conditions:

$$R' = 0.25 \text{ cm}$$
$$K_v' = 0.018 \text{ cp}$$
$$\rho' = 0.00130 \text{ gm/cm}^3$$
$$C_p' = 0.232 \text{ cal/gm } ^\circ\text{K}$$
$$k' = 5.22 \times 10^{-5} \text{ cal/cm sec } ^\circ\text{K}$$
$$K_T' = 1.23 \text{ sec}^{-1}$$
$$D' = 10^{-4} \text{ gm/cm sec}$$
$$G' = 0.0282 \text{ gm/sec (Mass Flow Rate)}$$
$$T_o' = 300^\circ \text{ K}$$
$$\Delta\hat{H}' = -42 \text{ cal/gm}_A \text{ reacted}$$
$$w_o' = 0.5 \text{ gm}_A\text{/gm}$$
$$\Delta H_a' = 8350 \text{ cal/gm mol } ^\circ\text{K}$$

Calculate the length of tube required for 80 per cent conversion and the maximum temperature occurring in the tube.

From these properties other quantities of interest are calculated.

$$U_o' = \frac{G'}{\pi R'^2 \rho'} = \frac{0.0282}{(0.25)^2(0.00130)}$$

$$= 110.5 \text{ cm/sec}$$

$$Re = \frac{(2R')U'\rho'}{K_v'} = \frac{(0.50)(110.5)(0.00130)}{(0.00018)}$$

$$= 400$$

$$Pr = \frac{C_p' \, K_v'}{k'} = \frac{(0.232)(0.00018)}{(0.0000522)}$$

$$= 0.8$$

$$Sc = \frac{K_v'}{D'} = \frac{(0.00018)}{(0.00001)}$$

$$= 1.8$$

$$\alpha = \frac{K_r' \, R'^2 \, \rho'}{4 \, D'} = \frac{(1.23)(0.25)^2(0.00130)}{4(0.0001)}$$

$$= 0.25$$

$$\Delta\hat{H} = \frac{\Delta H' \, w_o'}{C_p' \, T_o'} = \frac{(-42)(0.5)}{(0.232)(300)}$$

$$= -0.30$$

$$E_a = \frac{\Delta H_a'}{R' \, T_o'} = \frac{8350}{(1.987)(300)}$$

$$= 14$$

For 80 per cent conversion $Z^* = 0.48$ from Table 25.

Since

$$Z^* = 16\left(\frac{1+n}{1+3n}\right)\frac{z'}{2R'Re}\;\frac{\alpha}{Sc}$$

$$z' = \frac{2R'Re}{8}\;\frac{Sc}{\alpha}\,Z^* = \frac{(0.50)(400)(1.8)(0.48)}{8(0.8)}$$

$$= 27.2 \text{ cm}$$

The maximum temperature to be expected is found from Table 25 to be about $T = 1.11$

$$\text{or } T' = 1.11(300)$$

$$= 333^\circ \text{ K}$$

2. To reduce heat effects, the tube size is reduced to

$$R' = 0.111 \text{ cm}$$

Calculate the length of tube required and the maximum temperature occurring in the tube.

Then

$$\alpha = 0.050$$

$$Re = 2{,}000$$

and from Table 25 $\quad Z^* \doteq 0.66 \quad$ and $T \doteq 1.03$

$$z' = \frac{(0.111)(2{,}000)(1.8)(0.48)}{8(0.8)}$$

$$= 300 \text{ cm}$$

$$T' = (1.03)(300)$$

$$= 309^\circ \text{ K}$$

3. When the values of $\alpha Pr/Sc$, $\Delta\hat{H}$ and E_a are not equal to values

found in the tables, interpolation is required. Interpolation within a table for values of $\alpha Pr/Sc$ is straightforward. Interpolation between tables is illustrated below. For example, the length of tube (Z^*) is desired for 80 per cent conversion for a reaction with the following properties:

$$\alpha Pr/Sc = 0.25$$

$$\Delta\hat{H} = 0.2$$

$$E_v = 18$$

From Tables 24, 25, 26, 27, and 28 the following information is obtained.

$\Delta\hat{H}$	E_v	Z^*
-0.1	14	0.72
-0.3	14	0.48
-0.5	14	0.25
-0.1	21	0.66
-0.3	21	0.26

Isothermal Flow

$$Z^* = 0.81$$

These values are plotted in Figure 24 from which an interpolated value for $\Delta\hat{H} = -0.2$, $E_a = 18$ is obtained to be

$$Z^* \doteq 0.53$$

4. A non-Newtonian slurry containing a reactive material is flowing in a tubular reactor with the following properties:

$$R' = 0.50 \text{ cm}$$

$$K_v' = 31.2 \text{ gm/cm sec}^{2-n}$$

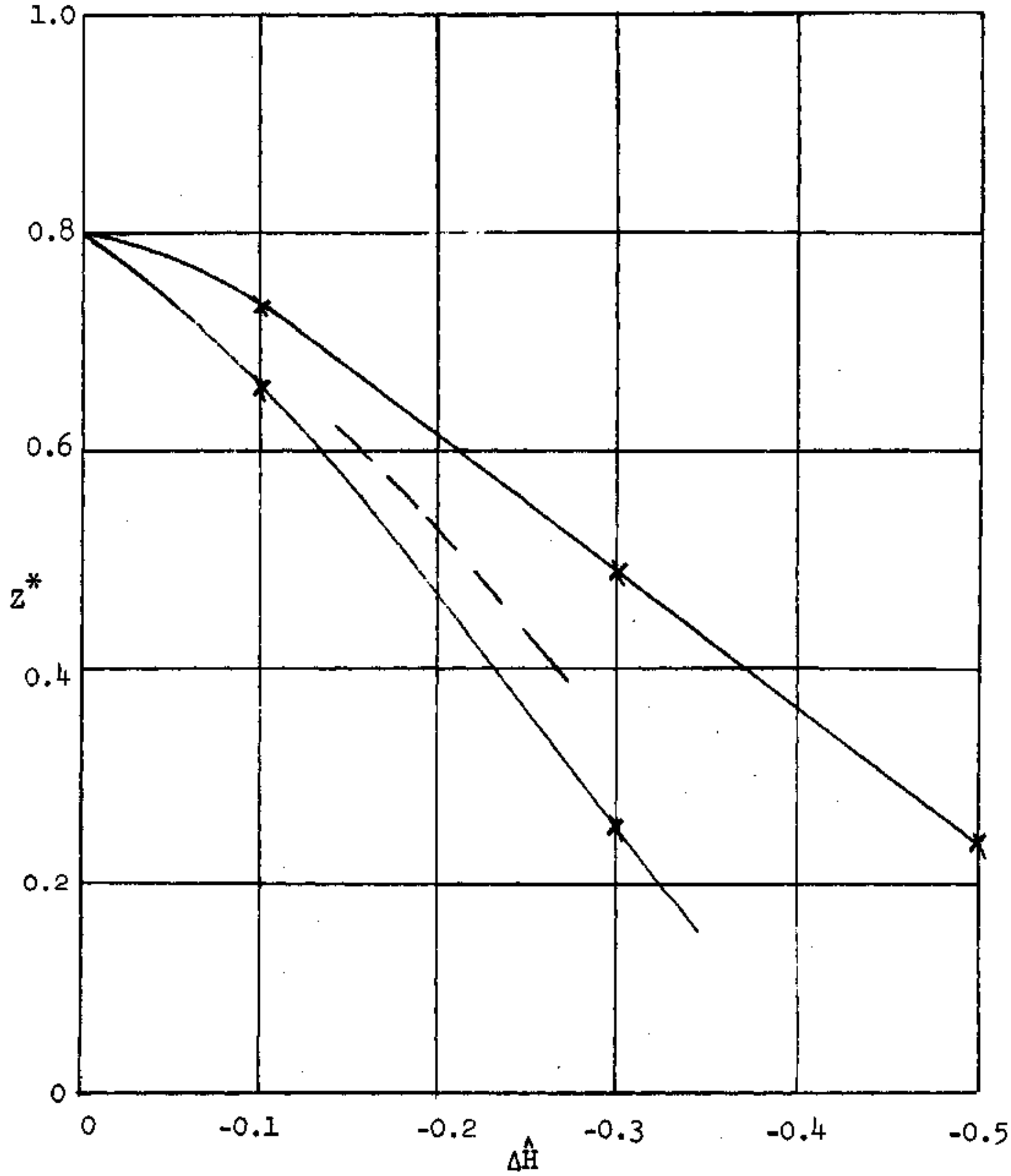


Figure 24. Illustration of Interpolation in Sample Calculation.

$$n = 0.554$$

$$\rho' = 1 \text{ gm/cm}^3$$

$$k' = 0.0015 \text{ cal/cm sec K}$$

$$C_p' = 1 \text{ cal/gm K}$$

$$D' = 1 \times 10^{-5} \text{ gm/cm sec}^{2-n}$$

$$K_r' = 0.24 \text{ sec}^{-1}$$

$$G' = 2680 \text{ gm/sec}$$

$$T_o' = 300^\circ \text{ K}$$

$$\Delta H' = -900 \text{ cal/gm reacted}$$

$$W_o' = 0.1 \text{ gm}_a\text{/gm}$$

$$\Delta H_a = 12550 \text{ cal/gm mole } ^\circ\text{K}$$

Calculate the length of tube required and the maximum temperature occurring in the reactor.

From these properties other quantities are calculated.

$$U_o' = \frac{G'}{\pi R'^2 \rho'} = \frac{(2680)}{\pi (0.50)^2 (1)}$$

$$= 1290 \text{ cm/sec}$$

$$Re = \frac{(2R)^n U_o'^{2-n} \rho'}{K_v'} = \frac{(1)^{0.554} (1290)^{1.446} (1)}{31.2}$$

$$= 1000$$

$$Pr = \frac{C_p' K_v'}{k'} \left(\frac{2R'}{U'}\right)^{1-n} = \frac{(1)(31.2)}{(0.0015)} \left(\frac{1}{1290}\right)^{0.446}$$

$$= 400$$

$$Sc = \frac{K_v'}{D'} \left(\frac{2R'}{U'}\right)^{1-n} = \frac{(31.2)}{(0.00001)} \left(\frac{1}{1290}\right)^{0.446}$$

$$= 60{,}000$$

$$\alpha = \frac{K_r' \, R'^2 \, \rho'}{4D'} = \frac{(24)(0.50)^2(1)}{4(0.00001)}$$

$$= 1500$$

$$\Delta H = \frac{\Delta \hat{H}' \, w_o'}{C_p' \, T_o'} = \frac{(-900)(0.1)}{(1)(300)}$$

$$= -0.3$$

$$E_a = \frac{\Delta H_a'}{R' \, T_o'} = \frac{12,550}{(1.987)(300)}$$

$$= 21$$

From Table 51 for 80 per cent conversion

$$Z^* \doteq 0.183$$

and

$$T \doteq 1.306$$

Then

$$z' = \frac{(1+3n)}{16(1+n)} \; \frac{ReSc}{\alpha} \; 2R' \, Z^*$$

$$= \frac{(1+3(0.554))}{16(1 + 0.554)} \; \frac{(1000)(60,000)}{1500} \; (1)(0.183)$$

$$= 785 \text{ cm}$$

$$T' = (1.306)(300)$$

$$= 392^\circ \text{ K}$$

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He entered the Georgia Institute of Technology to study on the co-operative plan in September of 1955, and received the degree of Bachelor of Chemical Engineering (with Honor) in June of 1960. His industrial employer during this time was E. I. du Pont de Nemours and Company in Richmond, Virginia.

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In August, 1960, he married the former Carolyn Delaine Bennett of Asheville, North Carolina. They have one son, John Bennett Whatley.